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Modelling of Nonlinear Torsional-Flexural Vibrations of Drill Strings

Abstract. Nonlinear flexural-torsional vibrations of shallow drill-strings are investigated. The drill-string is represented as an elastic rod rotating at constant angular velocity under the action of the longitudinal compressive force. The nonlinear model of flexural and torsional vibrations of drill-string is constructed on the basis of the theory of finite deformations of V.V. Novozhilov and its second system of simplifications. The numerical analysis of the model is carried out in the environment of symbolic mathematical computations – Wolfram Mathematica. The dominance of flexural vibrations of the drill-string over torsional vibrations is established. The influence of drill-string parameters on its oscillatory processes is investigated. It will allow to build up the modes of the drill-string movements to improve the quality of drilling of wells.

Key words: drill-string, dynamics, torsional-flexural vibrations, finite deformations, nonlinear model.

Introduction

Nowadays intensive development of the Earth's interior is characterized by the growth of oil and natural gas production. Construction of vertical wells by drilling is the most widespread way of production of oil products in countries with developed mining industry. It is safe and effective in a variety of geological conditions. However, the practice of construction of oil and gas wells shows that there are cases when there is a curvature of the vertical wellbore, which jeopardizes the possibility of its use. The appearance of emergency situations caused by critical states of quasi-static equilibrium and vibrations of the drill-string can serve as the reasons of wells deviation, as well as the bracking of a well due to the rotation of the drill-string, which generates as a result the centrifugal and Coriolis forces of inertia. To resolve these problems with the wells it is necessary to increase the efficiency of their drilling avoiding the buckling movements of the drill-string. Therefore, modelling of the drill-string dynamics and identification of safe modes of wells drilling represent a significant scientific and practical interest.

The study of the flexural-torsional vibrations as typical type of deformation of compressed and twisted drill-strings is one of the problems of the drilling equipment dynamics. Flexural-torsional vibrations were studied in linear statement in the majority of works on dynamics of elastic rod elements. For example in [1, 2] the linear theory of propagation of flexural-torsional vibrations in thin elastic beam of arbitrary section is considered. In [3] flexural-torsional vibrations of continuous beams and frames with distributed parameters were studied in linear statement [4] studies curving of the rod with a linear function of the curvature of all varieties of bending. S.P. Timoshenko [5, 6] obtained the linear equation of flexural-torsional vibrations of straight nontwisted rods with asymmetrical cross-section. In [7, 8] linear equations of flexural, flexural-torsional, flexural-longitudinal vibrations of twisted rotating rods taking into account the cross-sectional warping shear and torsion were obtained.

Often there are not enough classical linear theories and it is necessary to consider theories of higher approximations, in particular, considering geometrical and physical nonlinearity. And only a few papers devoted to the problems of torsion of rods in the nonlinear formulation. Thus, in [9] the nonlinear flexural–flexural-torsional vibrations of twisted rods, described by a system of three nonlinear integral-differential equations in partial derivatives, were investigated. The equation takes into account warping of the cross-section of the rod. However, the sampling system of vibrations are presented in a series of its own forms of the linear problem. In [10] in the framework of geometrically nonlinear model of elastic rod the interactions of flexural and torsional waves, resulting in the formation of periodic and solitary stationary waves, were studied, but only one-dimensional transverse vibrations were considered. Also, nonlinear models of flexural-torsional vibrations of an elastic rod were considered in [11, 12].

The problem of vibrations occurring in compressed and twisted rods with no restrictions on the size of their deformations is poorly studied, so this fact makes this problem actual and represents a scientific and practical interest to the oil and gas industry.

In this regard, this paper investigates the flexural-torsional vibrations of shallow drill-strings with no restrictions on the sizes of deformations. The nonlinear theory of finite deformations of V. V. Novozhilov [13] was used to derive the equations of drill-string movement.

Problem Statement

The drill-string is considered as an isotropic rod of length *l*, having a circular cross-section. With such cross-sectional shapes their warping can be ignored even when the twist angles are finite [14]. The rod rotates with angular velocity ω and compressed by the longitudinal force *N(z,t)*. The system of equations, describing the interaction of flexural and torsional vibrations of a rod considering geometrical nonlinearity, was obtained on the basis of second system of simplifications of the nonlinear theory of finite deformations of V.V. Novozhilov [15]. According to the last, components of deformations and angles of rotation also rely of small sizes, but the angles of rotation of the second order are not neglected. The resulting system of equations has the form:

$$-\rho F\frac{\partial^2 u}{\partial t^2}+\rho F\omega^2 u-2\rho F\omega\frac{\partial v}{\partial t}-N(z,t)\frac{\partial^2 u}{\partial z^2}-EI_y\frac{\partial^4 u}{\partial z^4}+$$
$$+\frac{EF}{(1-\nu)}\frac{\partial}{\partial z}\left[\left(\frac{\partial u}{\partial z}\right)^3\right]+\frac{EI_y(11-6\nu)}{8(1-\nu)}\frac{\partial}{\partial z}\left[\frac{\partial u}{\partial z}\left(\frac{\partial\theta}{\partial z}\right)^2\right]-\frac{EI_y}{2(1-\nu)}\frac{\partial}{\partial z}\left[\frac{\partial v}{\partial z}\frac{\partial^2\theta}{\partial z^2}\right]-, \tag{1}$$
$$-\frac{EI_y}{(1-\nu)}\frac{\partial}{\partial z}\left[\frac{\partial^2 v}{\partial z^2}\frac{\partial\theta}{\partial z}\right]+\frac{EF(5-6\nu)}{2(1-\nu)}\frac{\partial}{\partial z}\left[\frac{\partial u}{\partial z}\left(\left(\frac{\partial v}{\partial z}\right)^2+\theta^2\right)\right]=0.$$

$$-\rho F\frac{\partial^2 v}{\partial t^2}+\rho F\omega^2 v+2\rho F\omega\frac{\partial u}{\partial t}-N(z,t)\frac{\partial^2 v}{\partial z^2}-EI_y\frac{\partial^4 v}{\partial z^4}+$$
$$+\frac{EF}{(1-\nu)}\frac{\partial}{\partial z}\left[\left(\frac{\partial v}{\partial z}\right)^3\right]+\frac{EI_y(11-6\nu)}{8(1-\nu)}\frac{\partial}{\partial z}\left[\frac{\partial v}{\partial z}\left(\frac{\partial\theta}{\partial z}\right)^2\right]+\frac{EI_y}{2(1-\nu)}\frac{\partial}{\partial z}\left[\frac{\partial u}{\partial z}\frac{\partial^2\theta}{\partial z^2}\right]+ \tag{2}$$
$$+\frac{EI_y}{(1-\nu)}\frac{\partial}{\partial z}\left[\frac{\partial^2 u}{\partial z^2}\frac{\partial\theta}{\partial z}\right]+\frac{EF(5-6\nu)}{2(1-\nu)}\frac{\partial}{\partial z}\left[\frac{\partial v}{\partial z}\left(\left(\frac{\partial u}{\partial z}\right)^2+\theta^2\right)\right]=0.$$

$$-\rho I_p\frac{\partial^2\theta}{\partial t^2}+\rho I_p\omega^2\theta+\frac{2EI_p}{(1+\nu)}\frac{\partial^2\theta}{\partial z^2}+\frac{(11-6\nu)EI_r}{96(1-\nu)}\frac{\partial}{\partial z}\left[\left(\frac{\partial\theta}{\partial z}\right)^3\right]-\frac{EF}{(1-\nu)}\theta^3+$$
$$+\frac{EI_p(11-6\nu)}{8(1-\nu)}\frac{\partial}{\partial z}\left[\frac{\partial\theta}{\partial z}\left(\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right)\right]+\frac{EI_y}{2(1-\nu)}\frac{\partial}{\partial z}\left[\frac{\partial^2 u}{\partial z^2}\frac{\partial v}{\partial z}-\frac{\partial u}{\partial z}\frac{\partial^2 v}{\partial z^2}\right]+ \tag{3}$$
$$+\frac{E(5-6\nu)}{4(1-\nu)}\left(\frac{1}{2}I_p\theta\left(\frac{\partial\theta}{\partial z}\right)^2+\frac{1}{2}I_p\theta^2\frac{\partial^2\theta}{\partial z^2}-2F\theta\left[\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right]\right).$$

with the following boundary conditions:

$$u(z,t)=v(z,t)=\theta(z,t)=0 \quad (z=0, z=l)$$
$$EI_y\frac{\partial^2 u(z,t)}{\partial z^2}=EI_y\frac{\partial^2 v(z,t)}{\partial z^2}=0 \quad (z=0, z=l) \tag{4}$$

and the initial conditions:

$$u(z,t)\big|_{t=0}=v(z,t)\big|_{t=0}=\theta(z,t)\big|_{t=0}=0$$
$$\left.\frac{\partial u(z,t)}{\partial t}\right|_{t=0}=C_1,\quad \left.\frac{\partial v(z,t)}{\partial t}\right|_{t=0}=C_2,\quad \left.\frac{\partial \theta(z,t)}{\partial t}\right|_{t=0}=0. \tag{5}$$

Here *u(z,t)* is transverse movement of particles of the median line of the rod in the *Oxz* plane; $v(z,t)$ is transverse movement of particles of the median line of the rod in the *Oyz* plane; $\theta(z,t)$ is the angle of rotation of the cross-section; C_1, C_2 are constants; ω is the angular velocity of rotation; $N(z,t)$ is the longitudinal compressive force; E is Young's modulus; ν is Poisson's ratio; F is the cross-sectional area of the rod; $I_y=\iint\limits_F x^2 dF$ is the axial moment of inertia; $I_p=\iint\limits_F (x^2+y^2)dF$ is the polar moment of inertia; $I_r=\iint\limits_F (x^4+y^4)dF$; l is the rod length.

Numerical Solution of Boundary Value Problem

The Bubnov-Galerkin method was used to determine the solution of (1)-(5). In [16] has been shown that this method allows to successfully analyze the behavior of drill-strings used for oil production in the vertical and deviated wells. In contrast to [17], a multimode approximation of the solution is considered here. It is assumed that a compressive load is constant and distributed along the length of rod, $N(z,t)=N$.

In accordance with the Bubnov-Galerkin method, components of transverse movement $u(z,t)$, $v(z,t)$ and angle of rotation of the rod cross-section $\theta(z,t)$ are presented in the form of series:

$$u(z,t)=\sum_{i=1}^{n} f_i(t)\sin\left(\frac{i\pi z}{l}\right), \tag{6}$$

$$v(z,t)=\sum_{i=1}^{n} h_i(t)\sin\left(\frac{i\pi z}{l}\right), \tag{7}$$

$$\theta(z,t)=\sum_{i=1}^{n} g_i(t)\sin\left(\frac{i\pi z}{l}\right). \tag{8}$$

The basis functions $\sin\left(\frac{i\pi z}{l}\right)$ are chosen so that they satisfy the boundary conditions (4). After substitution of the series (6) – (8) in equation (1) – (3) the nonlinear system of differential equations of the second order with respect to the unknown functions $f_i(t)$, $h_i(t)$, $g_i(t)$ is obtained. Moreover, it can be shown that the flexural-torsional vibrations will not appear on the even modes, so it makes sense to immediately present the components of movements and rotation angles of the cross-sections of elastic line of the drill-string by the method of Bubnov-Galerkin in the following form:

$$u(z,t)=\sum_{i=1}^{n} f_i(t)\sin\left(\frac{(2i-1)\pi z}{l}\right), \tag{9}$$

$$v(z,t)=\sum_{i=1}^{n} h_i(t)\sin\left(\frac{(2i-1)\pi z}{l}\right), \tag{10}$$

$$\theta(z,t)=\sum_{i=1}^{n} g_i(t)\sin\left(\frac{(2i-1)\pi z}{l}\right). \tag{11}$$

This approach allows to carry out a complete analysis of flexural and torsional vibrations of drill-strings without expenses of machine time for calculation of not arising harmonics, especially when a large number of modes is considered.

The implementation of the Bubnov-Galerkin method and the further numerical solution were made in the applied program for symbolic computation – Wolfram Mathematica 10. 2.

The stiffness switching method was applied to implement the numerical solution of the equations. This method includes two numerical methods:

1) eighth-order explicit Runge-Kutta method,
2) linear implicit Euler method.

Use of two numerical methods is caused by stiffness of the studied equations. The stiff equations (system of the equations) are understood as such tasks, where explicit methods do not work [18], i. e. using of explicit scheme at specified time integration steps leads to a sharp increase in the number of calculations, either a rapid increase of an error.

Thus, tasks can be stiff in one intervals and non-stiff in others [19]. While there is no stiffness in the system and numerical solution goes smoothly, the eighth-order explicit Runge-Kutta method is applied. As soon as the system becomes stiff, the implicit method of Euler automatically starts being used for the numerical solution.

This stiffness switching method is applied not only for the reason that investigated ODEs are stiff, but also in view of its high efficiency in comparison with other numerical methods [20].

Numerical Results

Numerical calculations of the model were carried out at the following values of parameters of the steel drill-string: $E = 2.1 \times 10^5 \, MPa$, $\rho = 7800 \; kg / m^3$, $\nu = 0.28$, outer diameter of the rod $D = 0.2m$, inner diameter $d = 0.12m$, $F = 2.01 \times 10^{-2} m^2$, $I_y = 6.84 \times 10^{-5} m^4$, $l = 500m$, $\omega = 5 \, rad/min$, $N = 2.2 \times 10^3$ N.

Considering the first four approximations of solution by the Bubnov-Galerkin method, amplitudes of flexural and torsional vibrations of middle section of drill-string (Figures 1-2) are determined.

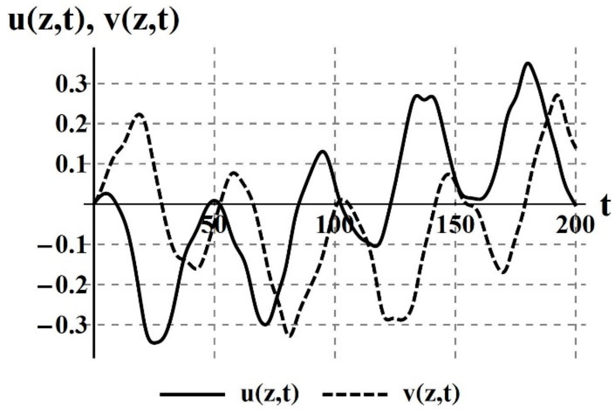

Figure 1 – Flexural vibrations of the drill-string by the first four harmonics

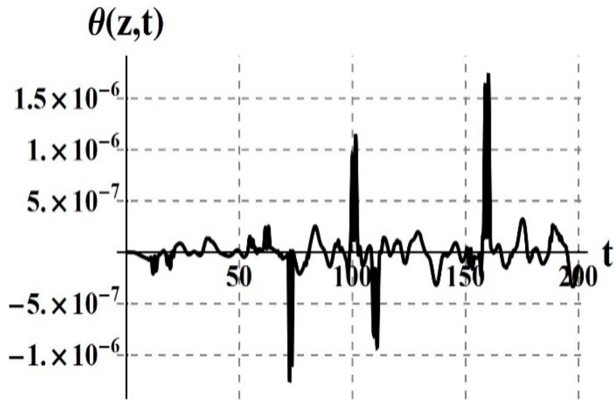

Figure 2 – Torsional vibrations of the drill-string by the first four harmonics

It can be seen from the analysis of results that flexural vibrations considerably surpass torsional vibrations. In this case, considering the type of initial and boundary conditions, torsional vibrations occur, appearing together with flexural vibrations. But the order is not so large. Nevertheless, influence of parameters of the drill-string on its flexural and torsional vibrations was studied. As these parameters were considered: length of the drill-string (Figures 3-5), rotational speed of the drill-string (Figures 6-8), radius and the thickness of its cross-section (Figures 9-11).

It is established that amplitudes of transverse vibrations with reduction of the length of drill-string are decreased, while amplitudes of torsional vibrations are increased. The calculations were carried out with the drill-string lengths: *l = 250 m*, *l = 500 m* (Figures 3-5). Figures 6-8 show the effect of rotational speed of the drill-string ω on the flexural-torsional vibrations. For this purpose the length of the drill string is considered $l = 200\,m$, rotating with an angular velocity 1) ω = 15 *rad/min*, 2) ω = 30 *rad/mid*. Increasing the angular velocity from $\omega = 15$ *rad/min* to $\omega = 30$ *rad/min* doesn't involve increase of amplitude of transverse vibration, but, nevertheless, increases the frequency characteristic of oscillatory process. In the case of torsional vibrations decreasing of amplitude is observed, which indicates the stabilization of the drill-string dynamics.

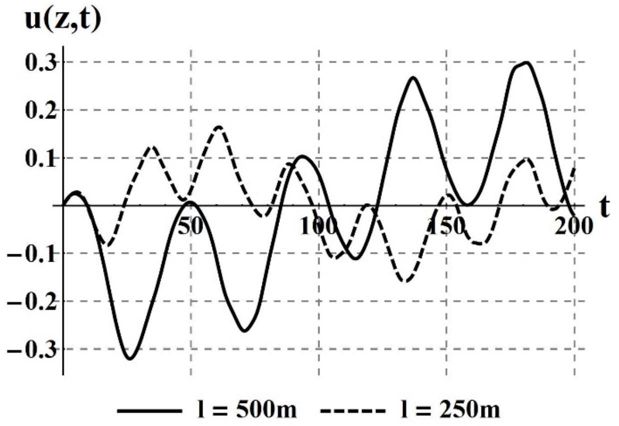

Figure 3 – Flexural vibrations $u(z,t)$ of different length drill-strings, *z = 0.5l*

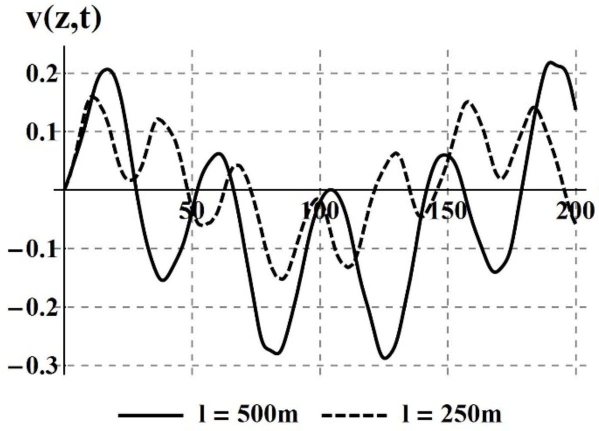

Figure 4 – Flexural vibrations $v(z,t)$ of different length drill-strings, *z = 0.5l*

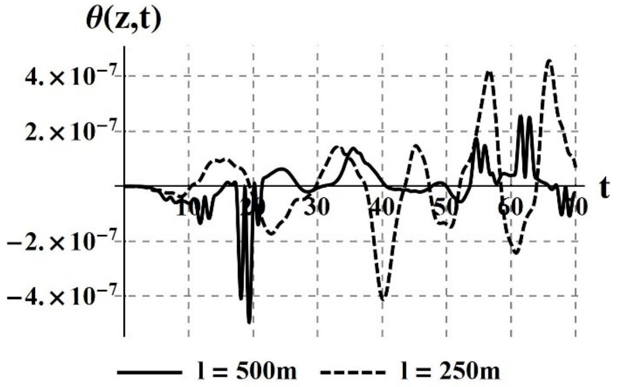

Figure 5 – Torsional vibrations $\theta(z,t)$ of different length drill-strings, *z = 0.5l*

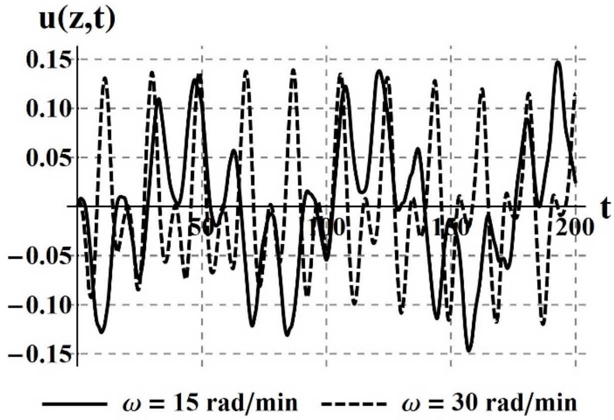

Figure 6 – Flexural vibrations $u(z,t)$ of drill-strings (*l = 200 m*) at different angular speeds

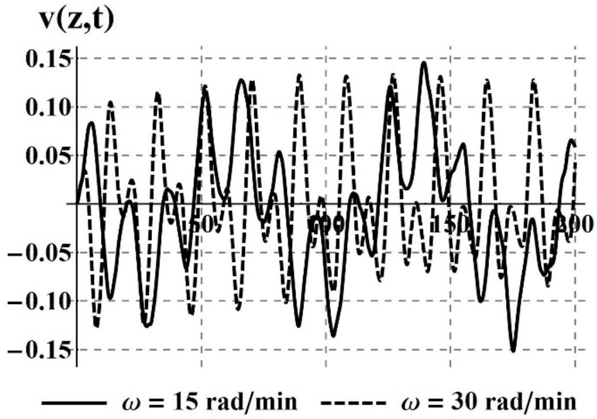

Figure 7 – Flexural vibrations $v(z,t)$ of drill-strings (*l = 200 m*) at different angular speeds

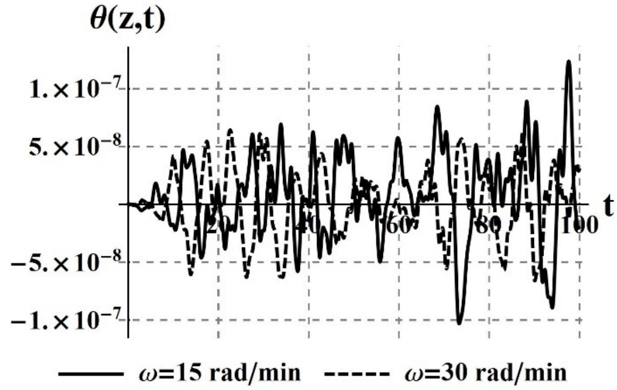

Figure 8 – Torsional vibrations $\theta(z,t)$ of drill-strings (*l = 200 m*) at different angular speeds

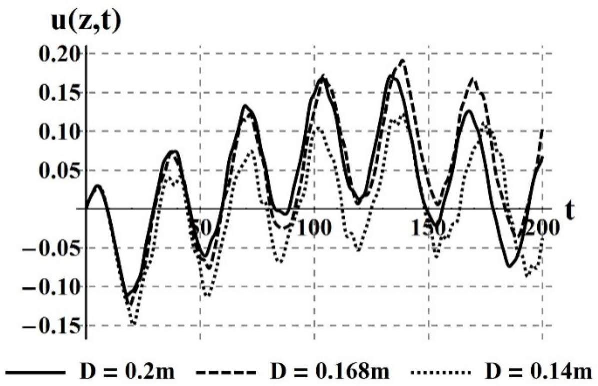

Figure 9 – Flexural vibrations $u(z,t)$ of drill-strings with different external diameter *l = 300 m* and $\omega = 5$ *rad/min*

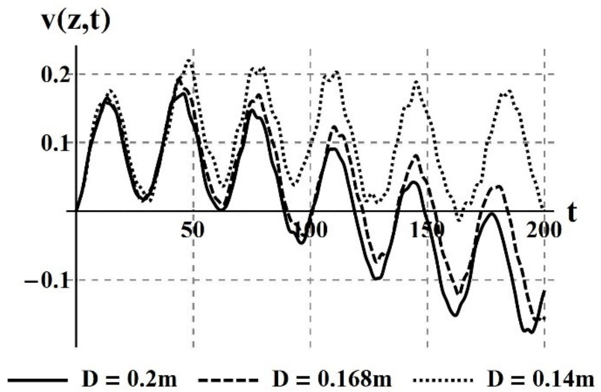

Figure 10 – Flexural vibrations $v(z,t)$ of drill-strings with different external diameter, *l = 300 m* and $\omega = 5$ *rad/min*

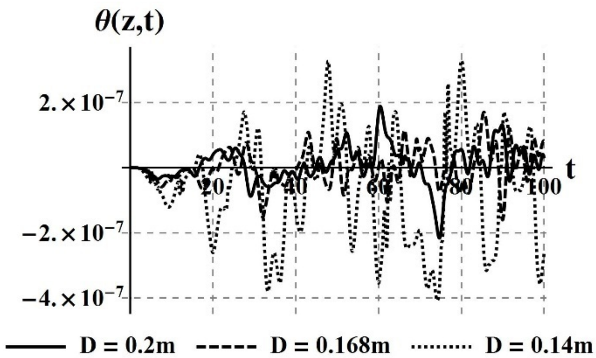

Figure 11 – Torsional vibrations $\theta(z,t)$ of drill-strings with different external diameter, *l = 300 m* and $\omega = 5$ *rad/min*

Influence of thickness of walls of the drill-sting on its flexural and torsional vibrations is investigated. Cases when sizes of external diameter of columns had the following values are considered: *D = 0.168 m* and *D = 0.14 m*.

It is established that reduction of external diameter of the drill-string from *D = 0.2 m* to *D = 0.168 m* (Figures 9-11), and, therefore, the thickness of the column, leads only to insignificant increase in amplitude of flexural and torsional vibrations. Reduction of external diameter to *D = 0.14 m* causes stronger indignation of vibrations.

Conclusion

In this paper the spatial movement of compressed and twisted drill-strings with no restrictions on the size of their deformations was modelled. The model is based on application of the Ostrogradsky-Hamilton variation principle and the theory of finite deformations of V. V. Novozhilov. The numerical analysis of nonlinear mathematical model of flexural-torsional vibrations of the drill-string for a spatial case was carried out. The calculation procedure of the mathematical model was developed and implemented in the environment of Wolfram Mathematica 10. 2. This procedure based on applying the Bubnov-Galerkin method for converting the original system of nonlinear equations with distributed parameters in a system of nonlinear ordinary differential equations and using a numerical stiffness switching method.

Results of researches testify that during the process of drilling a well in the drill-string can occur coupled flexural-torsional vibrations. Moreover, torsional vibrations is much less than flexural vibrations. Influence of parameters of the drill-string on its flexural and torsional vibrations is established. Increasing of the drill-string's length leads to an increase of the amplitude of flexural vibrations, while the amplitude of torsional vibrations decrease. Increasing the speed of rotation of the drill-string entails an increase of the frequency characteristics of the oscillatory process and reducing of the amplitude of torsional vibrations that testifies to stabilization of the drill-string movement. Reducing of the outer diameter of the drill-string, and accordingly, the thickness of its walls significantly influences only on the amplitude of torsional vibrations. However, their order is much less than values of the amplitude of flexural vibrations, which will allow to neglect this fact and to save the costs by manufacturing of drill-strings with smaller external diameter.

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