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Protoplanetary three-body problem with variable masses and its solutions

Abstract. Two protoplanetary three-body problem is considered in the case when the masses of all three bodies vary isotropically in different rates $\dot{m}_0 / m_0 \neq \dot{m}_1 / m_1$, $\dot{m}_0 / m_0 \neq \dot{m}_2 / m_2$, $\dot{m}_1 / m_1 \neq \dot{m}_2 / m_2$. It is assumed that the mass of the protoplanets much less than the mass of the proto-Sun $m_1(t) << m_0(t)$, $m_2(t) << m_0(t)$. Laws of change masses of bodies are known. The bodies are assumed as material points. On the basis of motion equations in the Jacobi coordinates the problem is described in the analogues of the Poincare second system. The perturbation function is symbolically computed in the system of the analytical calculations Mathematica up to the second degree of small quantities e_1, e_2, i_1, i_2 inclusive. It is obtained the equations of secular perturbations and its solutions for the protoplanetary three-body problem with masses changing isotropically in different rates.
Keywords: three-body problem, secular perturbations, protoplanets, variable masses, analogues of the Poincare second system

Introduction

Everyone knows that the cosmic bodies are unsteady. Over time they change the mass, size, shape and structure of the mass distribution within the body. These processes often occur in binary and multiple systems. In this regard, we are considered the problem of the three bodies with variable masses which had been investigated by the methods of perturbation theory [1-4].

The formulation of problem. The equations of motion in the coordinate system of Jacobi

Under consideration a system of mutually gravitating three bodies: T_0 **is** protosun with variable mass $m_0 = m_0(t)$, T_1 is internal protoplanet with variable mass $m_1 = m_1(t)$ and T_2 is external protoplanet with variable mass $m_2 = m_2(t)$. Fulfills the conditions

$$m_1(t) << m_0(t), \qquad m_2(t) << m_0(t). \qquad (1)$$

The masses of the bodies vary isotropically in different rates

$$\frac{\dot{m}_0}{m_0} \neq \frac{\dot{m}_1}{m_1}, \qquad \frac{\dot{m}_0}{m_0} \neq \frac{\dot{m}_2}{m_2}, \qquad \frac{\dot{m}_1}{m_1} \neq \frac{\dot{m}_2}{m_2}. \qquad (2)$$

The equations of motion in the Jacobian coordinates [1-5] can be written as

$$\begin{gathered} \mu_1 \ddot{\vec{r}}_1 = grad_{\vec{r}_1} U, \\ \mu_2 \ddot{\vec{r}}_2 = grad_{\vec{r}_2} U - \mu_2 (2\dot{\nu}_1 \dot{\vec{r}}_1 + \ddot{\nu}_1 \vec{r}_1), \end{gathered} \qquad (3)$$

where

$$\mu_1 = \mu_1(t) = \frac{m_1 m_0}{m_0 + m_1} \neq const,$$

$$\mu_2 = \mu_2(t) = \frac{m_2 (m_1 + m_0)}{m_0 + m_1 + m_2} \neq const,$$

reduced masses and

$$U = f\left(\frac{m_0 m_1}{r_{01}} + \frac{m_0 m_2}{r_{02}} + \frac{m_1 m_2}{r_{12}}\right),$$

$$r_{01}^2 = x_1^2 + y_1^2 + z_1^2 = r_1^2,$$
$$r_2^2 = x_2^2 + y_2^2 + z_2^2,$$
$$r_1 / r_2 < 1,$$

$$r_{02}^2 = (x_2 + \nu_1 x_1)^2 + (y_2 + \nu_1 y_1)^2 + +(z_2 + \nu_1 z_1)^2,$$

$$r_{12}^2 = (x_2 - \nu_0 x_1)^2 + (y_2 - \nu_0 y_1)^2 + +(z_2 - \nu_0 z_1)^2,$$

$$\nu_1 = \nu_1(t) = \frac{m_1}{m_0 + m_1} \neq const,$$

$$\nu_0 = \nu_0(t) = \frac{m_0}{m_0 + m_1} \neq const,$$

f is gravitational constant. For the problem under study, on the basis of the equations of motion (3), is constructed the perturbation theory for aperiodic motion on quasiconic section [1].

The equations of motion in the analogues of the Poincare second system

The analogues of the canonical elements of the Poincare second system $\Lambda_i, \lambda_i, \xi_i, \eta_i, p_i, q_i$ are determined by the following formulas [5]

$$\begin{aligned}
&\Lambda_i = \widetilde{\beta}_i \sqrt{a_i}, && \lambda_i = n_i[\phi_i(t) - \phi_i(\tau)] + \pi_i = l_i + \Omega_i + \omega_i, && \\
&\xi_i = \sqrt{2\Lambda_i\left(1 - \sqrt{1 - e_i^2}\right)}\cos\pi_i, && \eta_i = -\sqrt{2\Lambda_i\left[1 - \sqrt{1 - e_i^2}\right]}\sin\pi_i, && \pi_i = \Omega_i + \omega_i, \\
&p_i = \sqrt{2\Lambda_i\sqrt{1 - e_i^2}(1 - \cos i_i)}\cos\Omega_i, && q_i = -\sqrt{2\Lambda_i\sqrt{1 - e_i^2}\left[1 - \cos i_i\right]}\sin\Omega_i. &&
\end{aligned} \tag{4}$$

where a_i, e_i, ω_i, Ω_i, i_i, $\phi_i(\tau_i)$ orbital elements are corresponding analogues of the Keplerian elements. In the non-resonant case the equation of secular perturbations in the analogues of the Poincare second system (4) have the form

$$\begin{aligned}
&\dot{\Lambda}_i = 0, && \dot{\xi}_i = \frac{\partial R^*_{i\sec}}{\partial \eta_i}, && \dot{p}_i = \frac{\partial R^*_{i\sec}}{\partial q_i}, \\
&\dot{\lambda}_i = -\frac{\partial R^*_{i\sec}}{\partial \Lambda_i}, && \dot{\eta}_i = -\frac{\partial R^*_{i\sec}}{\partial \xi_i}, && \dot{q}_i = -\frac{\partial R^*_{i\sec}}{\partial p_i},
\end{aligned} \tag{5}$$

where

$$R^*_{i\sec} = \frac{1}{\gamma_1^2(t)} \cdot \frac{\widetilde{\beta}_1^4}{2\mu_{10}\Lambda_1^2} + \frac{1}{\psi_1}\left[-\frac{b_1 r_1^2}{2} + f\left(\frac{m_0 m_2}{r_{02}} + \frac{m_1 m_2}{r_{12}} - \frac{m_2(m_0 + m_1)}{r_2}\right)\right]_{\sec},$$

$$R^*_{i\sec} = \frac{1}{\gamma_2^2(t)} \cdot \frac{\widetilde{\beta}_2^4}{2\mu_{20}\Lambda_2^2} + \frac{1}{\psi_2}\left[-\frac{b_2 r_2^2}{2} + f\left(\frac{m_0 m_2}{r_{02}} + \frac{m_1 m_2}{r_{12}} - \frac{m_2(m_0 + m_1)}{r_2}\right)\right]_{\sec} - \frac{1}{\psi_2} V_{\sec}, \tag{6}$$

$$V_{\sec} = \mu_2[(2\dot{\nu}_1\dot{x}_1 + \ddot{\nu}_1 x_1)x_2 + (2\dot{\nu}_1\dot{y}_1 + \ddot{\nu}_1 y_1)y_2 + (2\dot{\nu}_1\dot{z}_1 + \ddot{\nu}_1 z_1)z_2]_{\sec},$$

$$b_1 = b_1(t) = \mu_1 \frac{\ddot{\gamma}_1}{\gamma_1}, \quad \gamma_1 = \gamma_1(t) = \frac{m_0(t_0) + m_1(t_0)}{m_0(t) + m_1(t)},$$

$$b_2 = b_2(t) = \mu_2 \frac{\ddot{\gamma}_2}{\gamma_2}, \quad \gamma_2 = \gamma_2(t) = \frac{m_0(t_0) + m_1(t_0) + m_2(t_0)}{m_0(t) + m_1(t) + m_2(t)},$$

$$\widetilde{\beta}_1^2 = f \cdot \mu_1(t_0) m_1(t_0) m_0(t_0), \quad \widetilde{\beta}_2^2 = f \cdot \mu_2(t_0) m_2(t_0)[m_0(t_0) + m_1(t_0)], \qquad \psi_i = \psi_i(t) = \frac{\mu_i}{\mu_{i0}} = \frac{\mu_i(t)}{\mu_i(t_0)}.$$

It follows from equation (5) that the study of the equations of secular perturbations reduces to the solution of the next system

$$\dot{\xi}_i = \frac{\partial R^*_{i\sec}}{\partial \eta_i}, \qquad \dot{p}_i = \frac{\partial R^*_{i\sec}}{\partial q_i},$$
$$\dot{\eta}_i = -\frac{\partial R^*_{i\sec}}{\partial \xi_i}, \qquad \dot{q}_i = -\frac{\partial R^*_{i\sec}}{\partial p_i}, \tag{7}$$

where

$$\Lambda_1 = \Lambda_1(t) = \Lambda_1(t_0) = \widetilde{\beta}_1 \sqrt{a_1} = \sqrt{f\, \mu_1(t_0) m_1(t_0) m_0(t_0)}\sqrt{a_1(t_0)} = const,$$
$$\Lambda_2 = \Lambda_2(t) = \Lambda_2(t_0) = \widetilde{\beta}_2 \sqrt{a_2} = \sqrt{f\, \mu_2(t_0) m_2(t_0)\left[m_0(t_0) + m_1(t_0)\right]}\sqrt{a_2(t_0)} = const. \tag{8}$$

In this article we obtain the equation (7) in an explicit form.

Equations of the secular perturbations

For writing in expanded form the right side of equation (7) it is necessary to express the perturbing functions (6) in the analogues of the Poincare second system (4). In this article in expansion of perturbing function (6) the terms up to the second degree inclusive of small quantities $e_1,\, e_2,\, i_1,\, i_2$ had been saved [6].

The equations of secular perturbations for the eccentric elements $\xi_i,\, \eta_i$ have the form

$$\dot{\xi}_1 = \frac{\partial F^*_{1\sec}(\xi_i, \eta_i, t)}{\partial \eta_1}, \qquad \dot{\eta}_1 = -\frac{\partial F^*_{1\sec}(\xi_i, \eta_i, t)}{\partial \xi_1},$$
$$\dot{\xi}_2 = \frac{\partial F^*_{2\sec}(\xi_i, \eta_i, t)}{\partial \eta_2}, \qquad \dot{\eta}_2 = -\frac{\partial F^*_{2\sec}(\xi_i, \eta_i, t)}{\partial \xi_2}, \tag{9}$$

and the optical elements $p_i,\, q_i$ as follows

$$\dot{p}_1 = \frac{\partial F^*_{1\sec}(p_i, q_i, t)}{\partial q_1}, \qquad \dot{q}_1 = -\frac{\partial F^*_{1\sec}(p_i, q_i, t)}{\partial p_1},$$
$$\dot{p}_2 = \frac{\partial F^*_{2\sec}(p_i, q_i, t)}{\partial q_2}, \qquad \dot{q}_2 = -\frac{\partial F^*_{2\sec}(p_i, q_i, t)}{\partial p_2}, \tag{10}$$

where

$$F^*_{1\sec}(p_i, q_i, t) = \psi^*_1(t) F(p_i, q_i), \qquad F^*_{2\sec}(p_i, q_i, t) = \psi^*_2(t) F(p_i, q_i),$$

$$\psi^*_1(t) = -\frac{f m_1 m_2 \nu_0 B_1}{8\psi_1}, \qquad \psi^*_2(t) = -\frac{f m_1 m_2 \nu_0 B_1}{8\psi_2},$$

$$F(p_i, q_i) = K^*_1(p_1^2 + q_1^2) + K^*_2(p_2^2 + q_2^2) + K^*_3(p_1 p_2 + q_1 q_2), \tag{11}$$

$$K^*_1 = \frac{1}{\Lambda_1}, \quad K^*_2 = \frac{1}{\Lambda_2}, \quad K^*_3 = -\frac{2}{\sqrt{\Lambda_1 \Lambda_2}},$$

B_1 is Laplace coefficient. Solving of systems of differential equations of secular perturbations (9)-(10) can be obtained numerically and analytically [1-4].

In this article, according to the assumption, in expansion of the perturbing function we restrict to the second degree inclusive of small quantities e_i, i_i. Therefore, also restricting up to the second degree inclusive of small quantities e_i, i_i in the formulas (11), we have

$$p_i = \sqrt{\Lambda_i} \sin i_i \cos \Omega_i,$$
$$q_i = -\sqrt{\Lambda_i} \sin i_i \sin \Omega_i. \tag{12}$$

Hence we have

$$\Lambda_i \sin^2 i_i(t) = p_i^2 + q_i^2,$$
$$tg\Omega_i(t) = -q_i / p_i. \tag{13}$$

Analytical relations (12)-(13) completely describe the time variation of the orbital elements $i_i(t)$, $\Omega_i(t)$ for arbitrary changes of the masses laws (1)-(2).

Conclusion

The problem is studied in the analogues of the Poincare second system. With the help of analytical calculations system Mathematica for the protoplanetary three-body problem with masses changing isotropically in different rates the full expression of the perturbing function that consists of 684 terms was calculated.

On the basis of the perturbing function it was first obtained the equation of secular perturbations of the problem - non-autonomous canonical equations of secular perturbations.

The results of this work can be used to analyze the influence of the effects of masses variability on the dynamic evolution of non-stationary protoplanetary systems.

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