UDC 531, 539.2, 539.3, 539.4

*Bibossinov A.Zh., Iskakbayev A.I., Bekbauov B.E.

Faculty of Mechanics and Mathematics, al-Farabi Kazakh National University, Almaty, Kazakhstan
*e-mail:bibossinov@live.com

Research and simulation of plastic zone around the Griffith's crack

Abstract. In this paper, by using the structural strength criterion of Neuber – Novozhilov, a formula of the critical load for plane blunt and sharp cracks is obtained. According to the obtained formula modeling of sharp crack as a thin ellipse with a ratio of small and major semi axes one to ten is carried out. With application of a software package OpenFOAM, based on the finite volume methods, by creating a solver for small elastoplastic deformation, we found the shape and size of plastic zone around the blunt and sharp cracks. A size of plastic zone based on calculation results satisfactorily agrees with the experimental data in researches of Hahn and Rosenfield, and shape – with data of Tuba. The results allow evaluating the impact of the microstructure to ductile material and on the behavior of structures made from such materials.

Key words: Griffith's crack, plastic zone, stress concentration, elastoplastic deformation, finite volume method, OpenFOAM.

Introduction

In the development of technological processes of manufacturing and design elements with the specified functional properties of the material it is necessary to ensure the appropriate structure, which determines its mechanical and physical properties. In the structural mechanics of fundamental role is simulation of the interaction of various scales in the process of deformation and failure to improve the operational properties of product (durability, strength, fracture toughness) [1].

Influence of microdefects on the physical and mechanical properties of the material is studied by Irwin theory of strength, in which the defect is modeled by mathematical cut with ability to spread. In this case, there are singular points at the ends of the cut where stress tends to infinity by the asymptotic law. In Griffith's fracture mechanics study of the influence of defects on the properties of materials comes to a boundary value problem in plane deformation of a body having elliptical shape cutout.

The criteria for the crack form

For consideration of a defect in the form of an elliptical cutout it is necessary to determine the criteria for the limit of the ratio of the semiaxes of the ellipse. For this we consider a square plate with an elliptical defect in a generalized state of stress with the full tension. The symbols and loading scheme are shown in figure 1. Where L is a characteristic size of a square plate, a and b the lengths of the axes of the ellipse ($a>b$), and when $L/a \to \infty$ we arrive at the problem of the full tensile plane with an elliptic hole. Stress tensor components, σ_{yy}, along the x-axis can be written in the form [1]:

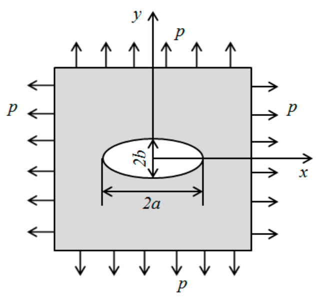


Figure 1 – The symbols and loading scheme of comprehensive stretching of the plate

$$\sigma_{yy} = px\left(x^2 - a^2 + 2b^2\right)\left(x^2 - a^2 + b^2\right)^{-3/2}, \quad (1)$$

where $p > 0$ is tensile force. We introduce the notations: $a = l$; $b = ml$; $0 \le m \le 1$.

We substitute the solution (1) into the structural strength criterion of Neuber–Novozhilov [2, 3, 4, 5]:

 Printed in Kazakhstan

$$\frac{1}{d}\int_{l}^{l+d}\sigma_{yy}(x)dx = \sigma_{ys}\,, \qquad (2)$$

where σ_{ys} is tensile strength of the material in tension (yield surface), d is the structural parameter of destruction having the dimension of length. After integration, we find the critical load, expressed in terms of m:

$$p_* = \sigma_{ys}\left[\left(m^2 l^2/d^2 + 2\,l/d + 1\right)^{1/2} - l/d\, m^2\left(m^2 + 2\,d/l + d^2/l^2\right)^{-1/2}\right]^{-1}. \qquad (3)$$

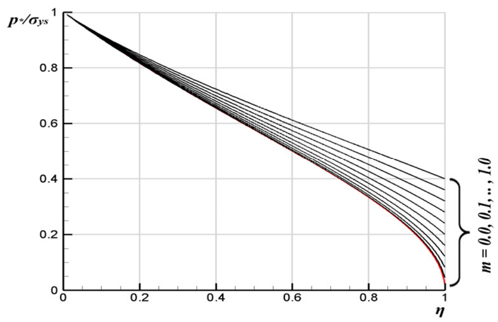
Equation (3) holds for blunt and sharp fractures [2]. According to this formula figure 2 shows the dependence of the critical stress on the crack length through the dimensionless variable $\eta=l/(l+d)$, for different values of ellipse axes relations in the range $0 \le m \le 1$.

If case of m = 0 we get the formula of the authors from [5] as:

$$\frac{p_*}{\sigma_{ys}} = \frac{1}{\sqrt{1+2l/d}} = \sqrt{\frac{1-\eta}{1+\eta}}\,, \qquad (4)$$

that derived for a uniaxial tensile elastic plate with straight crack length $2l$ (the same paper provides a comparison with experimental data).From (3) and (4) we obtain that the expression for the critical loads for small values of m is similar to cases for comprehensive and uniaxial tension.

Figure 2 shows that the critical loads for the case m = 0 and m = 0.1 have a little different. Therefore, the numerical analysis of elastic-plastic flows around the elliptical shape of the defect was performed for the case of m = 0.1.



		$m = 0$	$m = 0.1$	$m = 0.2$	$m = 0.3$	$m = 0.4$
$l/d = 10^{-2}$	$\eta = 0.01$	0.990	0.990	0.990	0.990	0.990
$l/d = 10^{-1}$	$\eta = 0.09$	0.913	0.913	0.913	0.914	0.915
$l/d = 10^{0}$	$\eta = 0.50$	0.577	0.578	0.581	0.586	0.592
$l/d = 10^{1}$	$\eta = 0.91$	0.218	0.222	0.232	0.248	0.268
$l/d = 10^{2}$	$\eta = 0.99$	0.070	0.081	0.106	0.139	0.174

p_* / σ_{ys}

Figure 2 – Dependence of the critical loads on the crack length for different values of the ratio of the semiaxes of the ellipse

Plastic zone simulation

We consider a square plate with an elliptical defect (a = 10b) in a unilateral stretch. The symbols and loading scheme are shown in figure 3.

Numerical analysis was conducted using the open source software package Open FOAM [6], by creating solver to small elastoplastic strains, using the Mises conditions in case of perfect plasticity.The increment of the plastic deformation tensor, $d\varepsilon_p$, depends on the stress: in lower limit only elastic deformations take place, $d\varepsilon_p = 0$; if achieve the limit value plastic deformations appear, the value of $d\varepsilon_p$ calculates from stress.

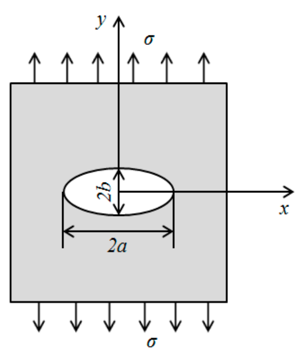


Figure 3 – Unilateral stretching of the plate. The symbols and loading scheme

The calculation results are presented in figure 4, which shows the curves for the dependence of the size of the zone of plasticity, r_p, for the plane strain and the plane stress.In the case of plane strain calculations were performed for different values of Poisson's ratio ($0.1 \le \nu \le 0.5$) for comparison with experimental data of plasticity zones from works [7, 8, 9].

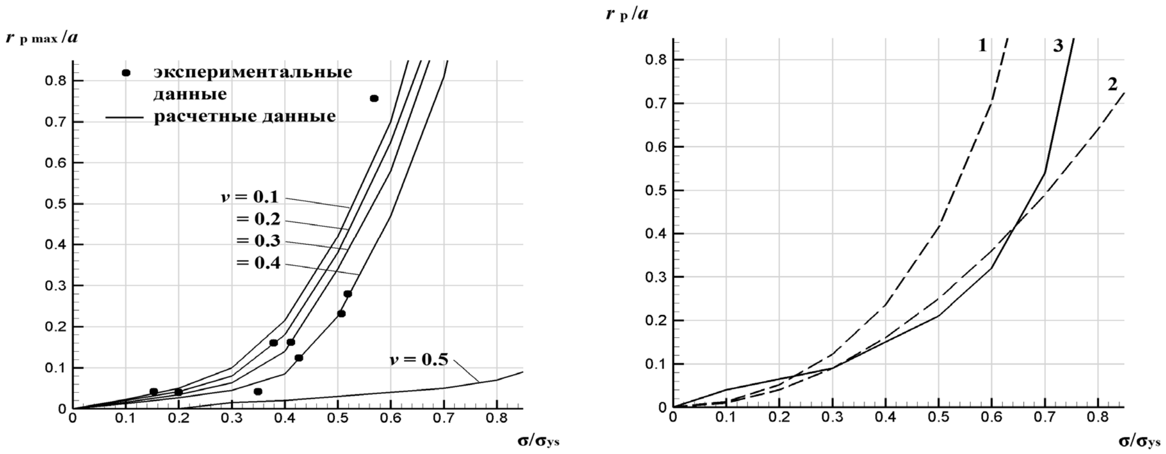


Figure 4 – Measured and calculated sizes of plasticity zone:
a – plain strain, the most distant point of the boundary zone of plasticity for different values of Poisson's ratio compared with the experimental data [7,9];
b – plain stress, plastic zone size along the *x* axis, curves 1 and 2 – by the Dugdale and Irwin formula [7, 8], 3 – numerical calculation

Researches of forms of plasticity zones made by Tuba, Rice and Rosengren, Hahn and Rosenfield are given in [7,8,9].By Tuba, the most distant point of the boundary zone of plasticity is at angle of 69°, as shown in figure 5 for different values of σ/σ_{ys}.

An important aspect is the experimental verification of the results of analytical calculations. Hahn and Rosenfield, from experimental research on the allocate region of plastic flow, came to the conclusion that none of the theoretical constructs are not fairly accurately describes a form of plasticity.Theoretical methods quite accurately describe the most distant point of the border areas of plasticity, but do not give an accurate assessment of the direction of the crack.From micropictures from Khan and Rosenfeld, in the case of plane stress, a form of plastic zone is most like the one that was presented Tuba, figure 5. [7,8]

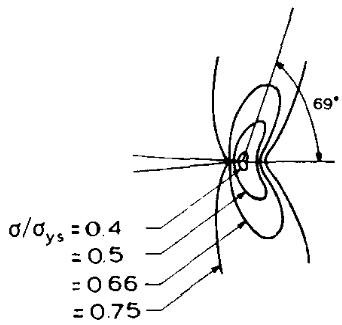


Figure 5 – The forms of plastic zones for fracture type I according Tuba [7, 8]

By numerical calculations we obtained forms of zone of plasticity (Figure 6) that consistent with the theoretical calculations made by Tuba. The numerical results (Figure 4 and Figure 6) describe the shape of plastic zone near the crack tip.

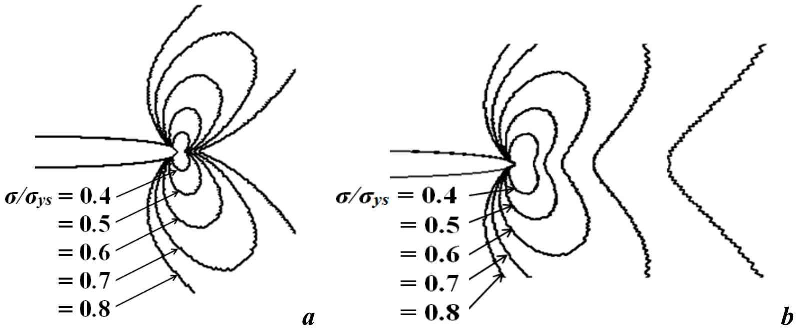


Figure 6 – Forms of plasticity zones based on the results of calculations:
a – plane strain; and *b* – plane stress

Conclusion

Since the theoretical studies do not give an accurate picture of the shape of plastic zone, and experimental studies have certain difficulties (experimentally difficult to distinguish between elastic and plastic deformation; measurements are performed on the sample surface), conducted numerical investigations are reasonable addition to research of zones of plasticity around the crack. Consideration of the cracks in the form of elliptical cutout simplifies the numerical implementation of the task. Modern computer technology makes it possible with the required accuracy to describe the process of elastoplastic flow around the crack of any shape.

References

1. Sedov L.I. Continuum Mechanics // M.: Nauka, 1984. –Vol. 2. – 560 p. (Rus.)

2. Kornev V.M. Hierarchy strength criteria of structured brittle environments// Applied Mechanics and Technical Physics. – 2000. – Vol. 41, No 2. – P. 177–187. (Rus.)

3. Mikhailov S.E. A functional approach to non-local strength conditions and fracture criteria. 1. Body and point fracture // Engineering Fracture Mechanics. – 1995. – Vol.52, No 4. – P.731-743.

4. Mikhailov S.E. A functional approach to non-local strength conditions and fracture criteria. 2. Discrete fracture // Engineering Fracture Mechanics. – 1995. – Vol.52, No 4. – P.745-754.

5. Petrov Y.V. Smirnov V.I. On the strength of materials with small defects // Solid Mechanics. – 2006. – Vol.4. –P.165-178. (Rus.)

6. http://www.openfoam.org/

7. Broek D. Elementary engineering fracture mechanics // Martinus Nijhoff Publishers, The Hague. – 1982. – 469 p.

8. Tuba I.S. A method of elastic-plastic plane stress and strain analysis // J. Strain Analysis. – 1966. –Vol.1. – P.115-122.

9. Hahn G.T., Rosenfield A.R. Plastic flow in the locale on notches and cracks in Fe–3Si steel under conditions approaching plane strain // Rep. to Ship structure Committee, 1968.