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Proton and electron fluxes on the surface of a dust grain in a plasma

Abstract. It is well known that the orbital motion limited approximation, being applied to dusty plasma, suggests that the electrons and ions of the plasma medium are independently absorbed by a dust grain. Moreover the classic version of the orbital motion limited approximation concedes that the interaction of the charged plasma particles with the dust is purely Coulomb, which means complete neglecting of the field screening phenomena. Such assumptions are only valid if the size of the dust particle is much smaller than the characteristic screening length, which in turn is much less than the mean free paths of electrons and ions in the plasma. In this case, application of the conservation laws of energy and angular momentum is enough to calculate the absorption cross section of electrons and ions by the dust particle. Herein an attempt is undertaken to treat the polarization of the dust particle itself in its interaction with the plasma particles, and it is demonstrated that the knowledge of the conservation laws of energy and angular momentum turns insufficient. It is shown that the absorption cross sections of electrons and ions of the plasma are determined by the condition that the kinetic energy of the incoming plasma particle should be equal to the maximum of the effective potential energy of their interaction with the dust grain. Ion and electron fluxes on the surface of a dust grain are calculated under the assumption that the corresponding velocity distributions are Maxwellian.

Key words: Dusty plasma, orbital motion limited approximation, polarization phenomena, dust charging.

Introduction

To understand all the physical properties of dusty plasmas it is necessary to determine the charge of the dust grains, which are macroscopic particles placed into the buffer plasma. To do this, one has to study the charging process of a dust particle due to fluxes of electrons and ions on its surface. For this purpose, the orbital motion limited approximation, first proposed by Langmuir and Mott-Smith[1], is widely used. Much later, Alpert et al. [2] and Laframboise [3] put forward the modern formulation of the method that allows one to predict the charge of the dust particles of different sizes [4-6], despite the simplifying assumption on the absence of collisions between plasma particles.

In the standard version of the orbital motion limited approximation the interaction of electrons and ions with the dust particle is described by either the Coulomb potential or the Debye-Huckelscreened potential. It is proposed in this paper, for the first time ever, to take into account the polarization phenomena, which provides an additional attraction mechanism between the plasma particles and the dust grain.

Dimensionless plasma parameters

The paper deals with the buffer plasma with the electron number density $n_в$ and the proton number density $n_p = n_в = n$, in which a spherical macroscopic particle of radius R and the electric charge $-Z_d e$ is placed Since the dust particle is solitary, it does not affect the neutrality of the buffer plasma, which leads to fulfillment of the condition $n_p = n_в = n$.

The state of the electron component of the plasma is described by the density parameter:

$$r_s = \frac{a}{a_B}, \tag{1}$$

where $a = (3/4\pi n)^{1/3}$ denotes the average distance between the electrons, $a_B = \hbar^2 / m_e e^2$ stands for the first Bohr radius with $\hbar$ being the Planck constant and e being the elementary electric charge.

Another dimensionless parameter relevant for the state of the buffer plasma is the so-called called coupling parameter given by:

$$\Gamma = \frac{e^2}{a k_B T}, \tag{2}$$

where k_B is the Boltzmann constant, T designates the ambient temperature. It should be noted that coupling parameter (2) is common to represent the ratio of the average Coulomb interaction energy of the electrons to their average kinetic energy of thermal motion.

To take into account the finite size of the dust particles, the size parameter is introduced as

$$D = \frac{a}{R}. \tag{3}$$

to show how many times the average distance between the buffer plasma particles is less than the radius of the dust particle.

Note that to determine the electric charge of the dust particle in the classical case it is sufficient to only know one dimensionless parameter

$$\Gamma_R = \frac{e^2}{Rk_BT} = D\Gamma . \tag{4}$$

Orbital motion limited approximation

Consider the interaction of a proton with a spherical dust particle, which is made of a conductive material. To account for the polarization effects of the dust grain, the potential energy of the interaction is written with the aid of charge image method as [7]:

$$U_{dp}(r) = -\frac{Z_d e^2}{r} - \frac{e^2 R^3}{2r^2(r^2 - R^2)}, \tag{5}$$

Let the dust particle absorb a proton with the fixed energy E and the impact parameter ρ. Itis known[8] that this process is governed by the effective potential energy defined as

$$U_{dp}^{eff}(r,\rho,E) = -\frac{Z_d e^2}{r} - \frac{e^2 R^3}{2r^2(r^2 - R^2)} + E\frac{\rho^2}{r^2}. \tag{6}$$

At the impact parameter $\rho = 0$, the effective potential energy of interaction betweena proton and a dust particle is a monotonically increasing function of the distance, it is negativeeverywhere and tends to $-\infty$ when the proton approaches the surface of the dust particle. This proton is obviously absorbed by the grain. At the fixed energy an increase in the impact parameter results in that a maximum appears in the curve of the effective potential energy whose height grows while increasing ρ. It is then evident that protons with small values of the impact parameter are absorbed by the dust particle, but at a certain value ρ_{dp} the height of the maximum of the effective potential energy turns equal to the total energy of the proton causing its rebound. Obviously, this value ρ_{dp} fully determines the absorption cross section as $\sigma_{dp} = \pi\rho_{dp}^2$. All above said is illustrated in Figure 1, which shows that the protons with the energy $E / k_BT = 1$ and the impact parameters $\rho = 0$ and $\rho = 1$ are absorbed by the dust particle, and those with the impact parameter $\rho = 2$ are scattered. The black line in Figure 1 corresponds to the value $\rho \approx 1.6$, which divides the region of the proton absorption and rebound with the energy $E / k_BT = 1$.

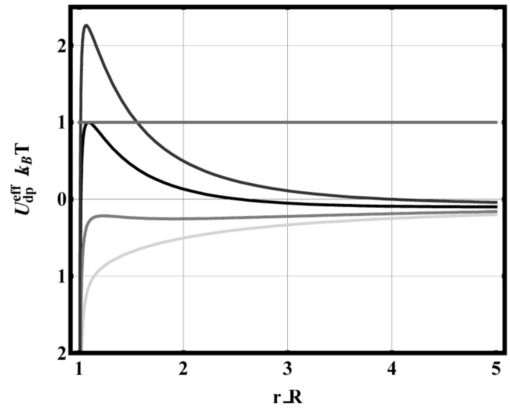

Figure 1 – The effective interaction energy between the proton and the dust particle at $E / k_BT = 1$, $\Gamma_R = 0.1$ and $Z_d = 10$. Green line: $\rho = 0$; magenta line: $\rho = 0$; black line: $\rho \approx 1.6$ that corresponds to the critical value at which the proton start to rebound from the dust grain; blue line: $\rho = 2$; red line: the total energy of the proton $E / k_BT = 1$

Thus, ρ_{dp} is obtained from the following condition:

$$\max U_{dp}^{eff}(r, \rho_{dp}, E)_{r \geq R} = E. \tag{7}$$

The numerical solution of equation (7) is foundas follows. For a fixed value of the energy E it is necessary to find such $\rho = \rho_{dp}$ that the maximum of effective potential energy (6) should exactly be equal to the total energy E.

Figures 2 and 3 show the dependence of the absorption cross section on the energy of the incident proton at different values of the dust particle charge and the coupling parameter. It is seen that the proton absorption cross section growswheneither the dust chargeorthe coupling parameter increase.

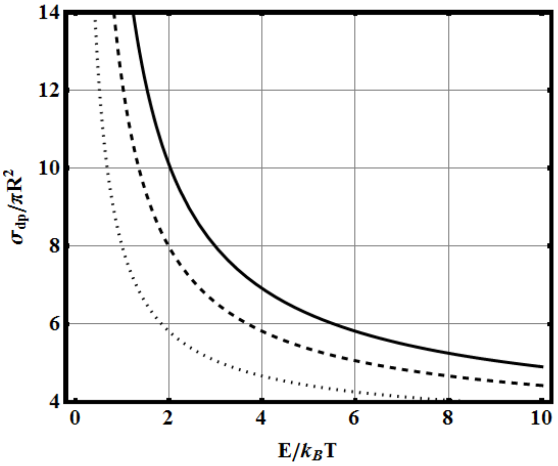

Figure 2 – The proton absorption cross section by the dust particle as a function of the energy of the incident proton at $Z_d = 10$. Dotted line: $Z_d = 5$; dashed line: $Z_d = 10$; solid line: $Z_d = 15$

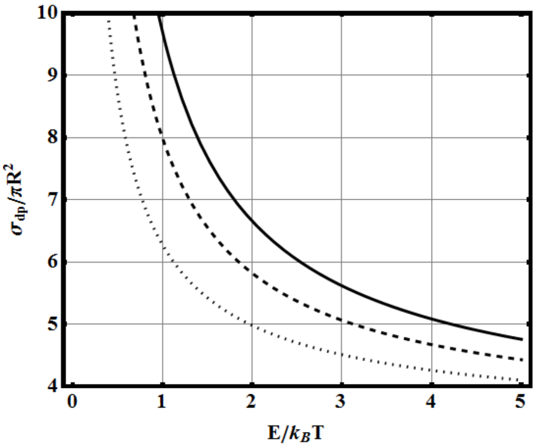

Figure 3 – The proton absorption cross section by the dust particle as a function of the energy of the incident proton at $\Gamma_R = 0.1$. Dotted line: $\Gamma_R = 0.1$; dashed line: $\Gamma_R = 0.2$; solid line: $\Gamma_R = 0.3$

Consider the interaction of an electron with the same spherical dust particle, which is made of a conductive material. The potential energy of the interaction is written with the aid of charge image method as [7]:

$$U_{de}(r) = \frac{Z_d e^2}{r} - \frac{e^2 R^3}{2r^2(r^2 - R^2)} . \qquad (8)$$

There is a significant difference for the interaction of the electron with the dust particle in comparison with its interaction with the proton. Due to the mutual repulsion the electron absorption is only possible when its energy reaches the critical value E_c determined as:

$$E_c = \max U_{de}(r) . \qquad (9)$$

Figures 4 and 5 show the dependence of the critical energy on the dust particle charge and the coupling parameter of the buffer plasma. It is clearly seen that both dependences are almost linear.

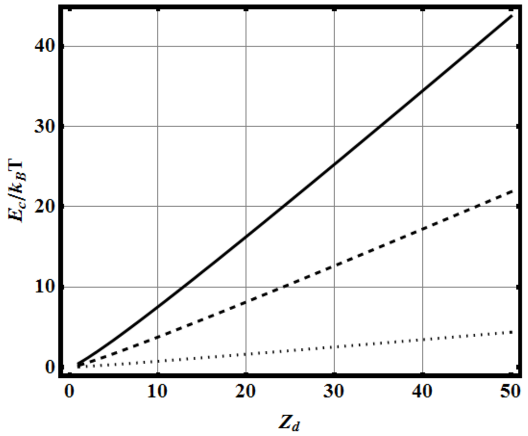

Figure 4 – The dependence of the critical electron energy on the dust particle charge. Dotted line: $\Gamma_R = 0.1$; dashed line: $\Gamma_R = 0.5$; solid line: $\Gamma_R = 1.0$

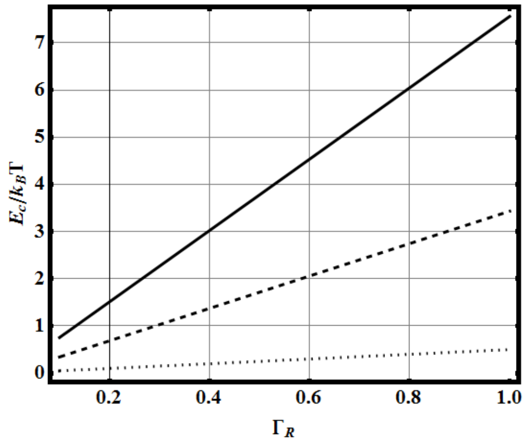

Figure 5 – The dependence of the critical electron energy on the coupling parameter. Dotted line: $Z_d = 1$; dashed line: $Z_d = 5$; solid line: $Z_d = 10$

Let the dust particle absorb a proton with the fixed energy E and the impact parameter ρ. Itis known [8] that this process is governed by the effective potential energy defined as

$$U_{dpe}^{eff}(r,\rho,E) = \frac{Z_d e^2}{r} - \frac{e^2 R^3}{2r^2(r^2 - R^2)} + E\frac{\rho^2}{r^2} . \qquad (10)$$

The analysis, quite an analogous to that made above for the proton absorption, remains almost unchanged for the interaction of the electron and the dust particle and the results are summarized in Figure 6. The only difference is that for the impact parameter $\rho = 0$, the effective interaction energy between the electron and grain already has a maximum which is the straightforward consequence of the appearance of the critical electron energy mentioned above.

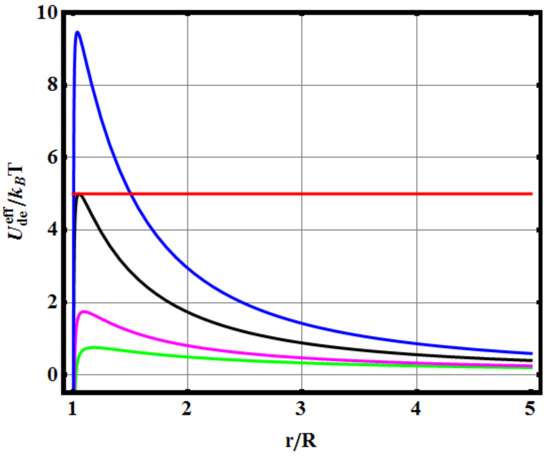

Figure 6 – The effective interaction energy between the electron and the dust particle at $E/k_BT = 1$, $\Gamma_R = 0.1$and $Z_d = 10$. Green line: $\rho = 0$; magenta line: $\rho = 0.5$; black line: $\rho \approx 1$ that corresponds to the critical value at which the electron starts to rebound from the dust grain; blue line: $\rho = 1.4$; red line: the total energy of the electron $E/k_BT = 1$.

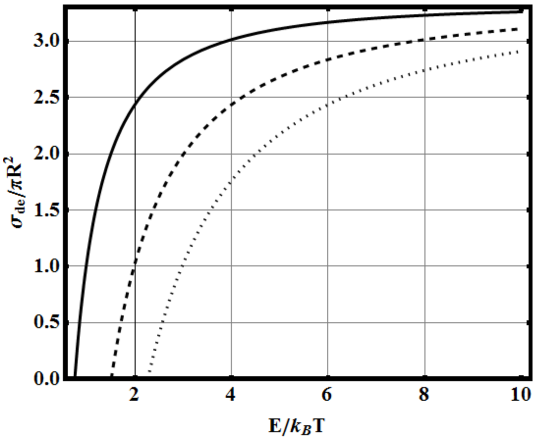

Figure 7 – The proton absorption cross section by the dust particle as a function of the energy of the incident proton at $Z_d = 10$. Dotted line: $\Gamma_R = 0.3$; dashed line: $\Gamma_R = 0.2$; solid line: $\Gamma_R = 0.1$.

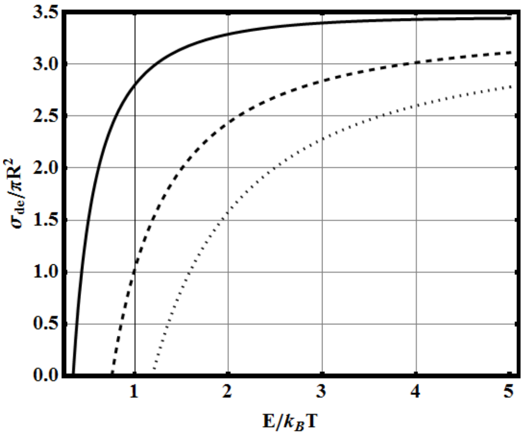

Figure 8 – The proton absorption cross section by the dust particle as a function of the energy of the incident proton at $\Gamma_R = 0.1$. Dotted line: $Z_d = 5$; dashed line: $Z_d = 10$; solid line: $Z_d = 15$

Figures 7 and 8visualize the dependence of the electron absorption cross section on the energy of the incident electron at different values of the dust particle charge and the coupling parameter. It is seen that the electron absorption cross section grows when either the dust charge decreases or the coupling parameter increases

Proton and electron fluxes

It is known that the proton flux on the surface of the dust particle is obtained from the relevant absorption cross section by integration over the velocity distribution function as:

$$J_p = n_p \int v\sigma_{dp} f_p(v) d\mathbf{v}\,, \tag{11}$$

where

$$f_p(v) = (2\pi v_{Tp}^2)^{-3/2} \exp\left(-\frac{v^2}{2v_{Tp}^2}\right), \tag{12}$$

and $v_{Tp} = \sqrt{k_BT/m_p}$ stands for the thermal velocity of protons with the mass m_p..

Figures 9 and 10 show the dependence of the proton flux on the particle surface as a function of its charge and the coupling parameter. It is seen that the proton flux grows when the charge of the dust particle and the coupling parameter increase due to reciprocal attraction. It is rather interesting to note that the corresponding dependences are almost linear.

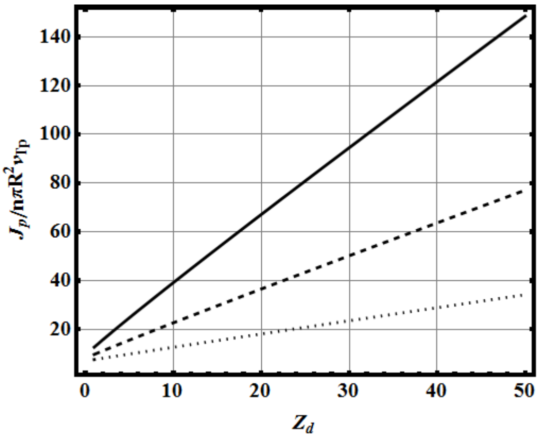

Figure 9 – The proton flux on the surface of the dust particle as a function of its charge. Dotted line: $\Gamma_R = 0.1$; dashed line: $\Gamma_R = 0.25$; solid line: $\Gamma_R = 0.5$

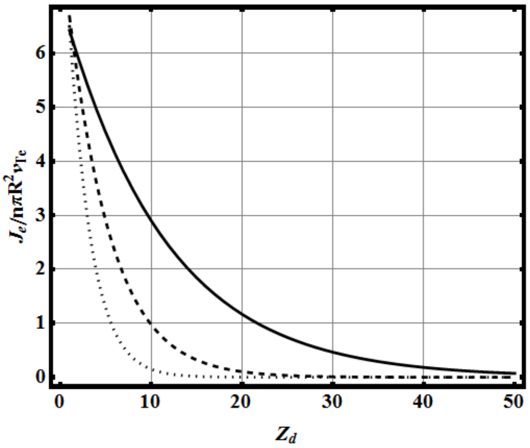

Figure 11 – The electron flux on the surface of the dust particle as a function of its charge. Dotted line: $\Gamma_R = 0.1$; dashed line: $\Gamma_R = 0.25$; solid line: $\Gamma_R = 0.5$

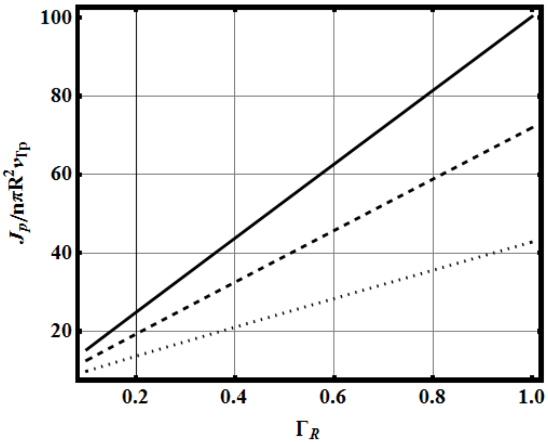

Figure 10 – The proton flux on the surface of the dust particle as a function ofthe coupling parameter Γ. Dotted line: $Z_d = 5$; dashed line: $Z_d = 10$; solid line: $Z_d = 15$

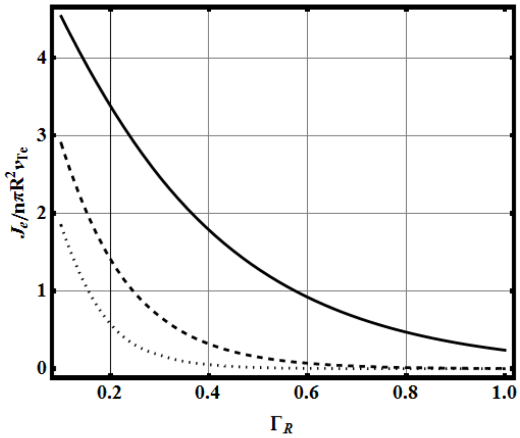

Figure 12 – The electron flux on the surface of the dust particle as a function ofthe coupling parameter Γ. Dotted line: $Z_d = 5$; dashed line: $Z_d = 10$; solid line: $Z_d = 15$

Similar to (11), the electron flux on the surface of the dust particle is determined by the absorption cross section as an integral over of the velocity distribution function

$$J_e = n_e \int v \sigma_{de} f_e(v) d\mathbf{v}, \qquad (13)$$

where

$$f_p(v) = (2\pi v_{Te}^2)^{-3/2} \exp\left(-\frac{v^2}{2v_{Te}^2}\right) \qquad (14)$$

and $v_{Te} = \sqrt{k_B T / m_e}$ stands for the thermal velocity of electrons with the mass m_e.

Figures 11 and 12demonstrate the dependence of the electron flux on the particle surface as a function of its charge and the coupling parameter. It is seen that the electron flux decreases when the charge of the dust particle and the coupling parameter grow due to reciprocal repulsion.

Conclusions

In this paper we have studied the proton and electron fluxes on the polarized dust particle immersed in the plasma. Consideration is entirely based on the orbital motion limited approximation, which implies the collision less ballistic trajectories of plasma particles in an electric field of the charged dust grain. It has been demonstrated that the polarization effects lead to a substantial modification of the calculation technique.

It is assumed that the dust particle is negatively charged resulting in the electron repulsion and proton attraction. As a result, the absorption of electrons by the dust particle can only occur when the electron energy reaches a certain value, which turns linearly dependent on the charge of dust particle and the coupling parameter. It has been found that the proton and electron fluxes on the grain surface strongly depend on its charge and the coupling parameter of the buffer plasma. In particular, the proton flux grows linearly with increasing the grain charge and the coupling parameter, which is explained by their mutual attraction. The opposite pattern is observed for the the electron flux since the electrons are repelled by the negatively charged dust particle.

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