International Journal of Mathematics and Physics 5, №1, 88 (2015)

UDC 539.12.01;539.12:530.131+539.125.17.

Tyulemissov Zh.Zh., Habyl N., Zhaugasheva S.A., *Nurbakova G.S., Saidullaeva G.G.

Al-Farabi Kazakh National University, Almaty, Kazakhstan
*e-mail: guliya_nurbakova@mail.ru

Significance of confinement in covariant quark model.

Abstract. We studied significance of confinment in covariant model of quarks. For this we studied analytical property of quarks' diagram's. Calculations were based on two point quark's loop with scalar quarks. We investigate ordinary local loop case and case with nonlocal crest factor with confiment and without it. To calculation real and imaginary part was used λ-method. We estimated matrix elements for that cases. It was shown that in case with cofinment, loop is real, smooth and without any cusp in the quarks-producing threshold.
The conclusion done in two ways, first, by the standard technique of Cauchy's theorem and analytic properties of the loop, and, secondly, using the original transformations of the integral into the form of the dispersion representation.
Key words: covariant quark model, infrared confinement, local quarks loop, non local quarks loop.

Introduction

Covariant quark model, which is used in this work based on the following physical picture. It is assumed that hadrons which are consist of quarks interact with each other through the exchange of virtual quarks. The fact that hadrons are bound states of quarks is taken into account by means of the so-called compositeness condition proposed by Weinberg [1] and Salam [2]. Physically, this condition means that the wave function renormalization constant of hadrons, which appeared as a result of interaction with quarks is set to zero. Hadron interaction among themselves and with the electromagnetic field and gauge fields of weak interactions are described by the quark diagrams. Production of colored quarks, i.e., the appearance of the relevant sections of unitary quark diagrams are forbidden by using the certain confinement mechanism, associated with the introduction of the infrared cutoff Schwinger parameter space.

The main objective ofthis paper is to study the analytic properties of the quark diagrams. All calculations are made on the example of the two-point quark loop with the scalar quarks. It was studied the cases of the common local loop, the loop with non-local form factor as without and with taking into account the confinement. For numerical computations of real and imaginary parts one used the original method proposed in [3] and called λ-method.

In this work we use the relativistic constituent quark model with confinement [4], the so-called covariant quark model.

1. Local quarks loop

In order to study the analytic properties of the quark diagrams, we start with the local case without vertex functions. In this case we have direct integration. For the ultraviolet convergence, it is necessary that the space dimension d < 4.

Let us consider integral corresponding to the two-point local loop:

$$J(p^2) = \int \frac{d^d k}{(2\pi)^d i} \frac{1}{[m^2-(k+p)^2][m^2-k^2]} = \int_0^1 dx \int \frac{d^d k}{(2\pi)^d i} \frac{1}{[m^2-xk^2-(1-x)(k+p)^2]^2} \quad (1)$$

After the shift and the integration over the angular coordinates of variables, we obtain

$$J(p^2) = \frac{2\pi^2}{\Gamma(d/2)} \frac{1}{(2\pi)^d} \int_0^1 dx \int_0^\infty dk \frac{k^{d-1}}{[A+k^2]^2} \tag{2}$$

where $A = m^2 - (1-x)^2$.

The obtained integral is calculated in standard way

$$J(p^2) = C(\omega)\Gamma(-\omega) \int_0^1 dx [m^2 - x(1-x)p^2]^\omega \tag{3}$$

where

$$\omega = d/2 - 2: \qquad C(\omega) = 1/(4\pi)^{\omega+2} = 1/4^{d/2}\pi^{d/2}$$

Note that the difference $J(p^2) - J(0)$ is finite for $d = 4$ or$\omega = 0$. Expanding this difference in ω, we obtain

$$\bar{J}(p^2) = J(p^2) - J(0) = C(\omega)\Gamma(-\omega) \int_0^1 dx\{1 + \omega \ln[m^2 - x(1-x)p^2] - 1 - \omega \ln m^2 + O(\omega^2)\} = C(\omega)\Gamma(-\omega) \int_0^1 dx \left\{\ln\left[1 - x(1-x)\frac{p^2}{m^2}\right] + O(\omega^2)\right\} \tag{4}$$

We take into account only the first terms of the expansion

$$\Pi_{\text{loc}}(p^2) = \lim_{\omega\to 0}(4\pi)^2 \bar{J}(p^2) = -\int_0^1 dx \ln[1 - x(1-x)\frac{p^2}{m^2}] \tag{5}$$

The integral is calculated exactly for negative values of the square of the momentum, and then make an analytic continuation for positive values.

$$\Pi_{\text{loc}}(p^2) = \begin{cases} \sigma(\ln[(\sigma-1)/(\sigma+1)] + i\pi) + 2, & p^2 < 0 \\ 2 + 2\sqrt{\sigma_1}\arctan\left(\sqrt{1/\sigma_1}\right), & 0 \le p^2 \le 4m^2 \\ \sigma(\ln[(1-\sigma)/(1+\sigma)] + i\pi) + 2, & p^2 > 4m^2 \end{cases} \tag{6}$$

By using the above formula we obtain the graph shown in figure 1.

Figure 1 shows the behavior of the square modulus of $\Pi_{\text{loc}}(p^2)$. There is cusp at the threshold of $4m^2$. The cusp is responsible for the decay of a particle on its constituents. In a number of papers which are devoted to solve the problem of confinement, one of fared" to drop" the imaginary part of the loop. The Fig.1 shows that the square of its real part seven more pronounced" *cusp*".That is such a naive approach does not solve the problem of confinement.

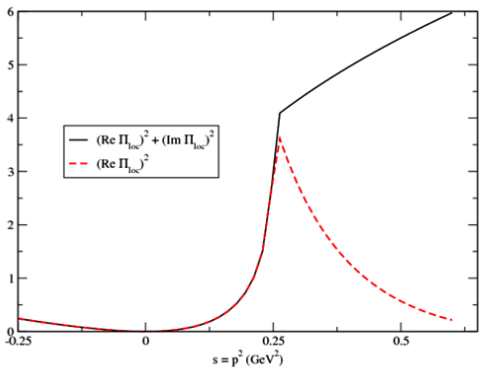

Figure 1 – The square of real part and the modulus of the matrix element

2. Infrared confinement.

Let us study an arbitrary Feynman diagram:

$$\Pi(p_1, \dots, p_m) = \int [d^4k]^l \prod_{i_1=1}^{n} S_{i_1}(k_{i_1}) \prod_{i_2=1}^{m} \Phi_{i_{2+n}}(-k_{i_2+n}^2) \quad (7)$$

Here l is the number of quark loop,m is the number of quark propagators, n is the number of vertex functions. To study the analytic properties of the diagram we will use the scalar quark propagator

$$\tilde{S}_q(k) = \frac{1}{m_q^2 - k^2} = \int_0^\infty d\alpha e^{-\alpha(m_q^2 - k^2)} \quad (8)$$

After integrating over the loop momentum variables the expression for the Feynman diagram can be written as

$$\Pi = \int_0^\infty d^n \, \alpha F(\alpha_1, \dots, \alpha_n) \quad (9)$$

In order to simplify the numerical integration move on to integration over the simplex, by inserting the one as

$$1 = \int_0^\infty dt \delta(t - \sum_{i=1}^{n} \alpha_i) \quad (10)$$

Producing the scaling variables $\alpha i \to t\alpha i$, we finally obtain

$$\Pi = \int_0^\infty dt t^{n-1} \int_0^1 d^n \alpha \delta(t - \sum_{i=1}^{n} \alpha_i) f(t\alpha_1, \dots, t\alpha_n) \quad (11)$$

One produces the (n-1) integration over the dimensionless parameter α, running through the simplex, and one integration over the parameter t, which has the dimension of the inverse square of the mass and ranges from zero to infinity. At occurrence of the branch threshold point this integral becomes converged at $t \to \infty$. If we cut the integration at the upper limit, it provides absence of any threshold singularities in this diagram because the integral obtained sabsolutely convergent for any set of kinematic variables. Therefore, the expression for an arbitrary diagram, taking into account the infrared quark confinement can be written as

$$\Pi_{conf} = \int_0^{1/\lambda^2} dt t^{n-1} \int_0^1 d^n \alpha \delta(t - \sum_{i=1}^{n} \alpha_i) f(t\alpha_1, \dots, t\alpha_n) \quad (12)$$

Here λ is a parameter of infrared cutoff. The case of $t \sim 0$, were the region of small values of the integration variable, corresponds to large values of loop momentum variables and is therefore called the ultraviolet region. In the covariant quark model ultraviolet divergences are absent due to the vertex functions that are rapidly decreasing in the Euclidean space.

3. Nonlocal quark loop.

Consider the case of nonlocal formfactor. Let us consider the integral corresponding to nonlocal two-point loop:

$$\tilde{F}(p^2) = \int \frac{d^4k}{(2\pi)^4 i} \exp[s_v k^2] \frac{1}{[m^2 - (k+p/2)^2][m^2 - (k-p/2)^2]} \quad (13)$$

We use the equation $1/A = \int d\alpha e^{-\alpha A}$, for A> 0 and introduce the notation $\Pi(p^2) = 16\pi^2 \tilde{F}(p^2)$

$$\tilde{\Pi}(p^2) = \iint_0^\infty d\alpha_1 \, d\alpha_2 \int \frac{d^4k}{\pi^2 i} \exp\left[s_v k^2 - \alpha_1 \left[m^2 - \left(k + \frac{p}{2}\right)^2\right] - \alpha_2 \left[m^2 - \left(k - \frac{p}{2}\right)^2\right]\right] \quad (14)$$

After integrating over k, we obtain

$$\widetilde{\Pi}(p^2) = -\int_0^\infty dt \frac{t}{(s_v^2+t^2)}(s_v^2+t^2-2s_v it)\int_0^1 dx \exp\left[-\epsilon t + \frac{s_v t^2}{s_v^2+t^2}\right] * \left[\cos\left[-tz_0 + \frac{{s_v}^2 t}{s_v^2+t^2}z_1\right] + +i\sin\left[-tz_0+\frac{{s_v}^2 t}{s_v^2+t^2}z_1\right]\right] \tag{15}$$

In this model the interaction of hadrons described by a set of relevant Feynman diagrams with local quark propagator and non-trivial vertex functions. As a consequence, the matrix elements of the physical processes occur threshold singularities corresponding to the appearance of quarks. In the space of Schwinger parameters one introduced the single dimensional parameter t, on which there is integration from zero to infinity. The appearance of threshold singularities corresponds to the divergence of the integral at infinity.

Infrared confinement provides absolute convergence of the integral and the absence of any singularities for any values of $\widetilde{\Pi}(p^2)$.

$$\widetilde{\Pi}(p^2) = \int_0^{1/\lambda^2} dt \int_0^1 dx \frac{t}{(s_v+t)^2}\exp\left[-tz_0 + \frac{s_v t}{s_v+t}z_1\right] \tag{16}$$

By using the above formula we obtain the graph shown in figure 2.

In figure 2 is shown the square of the real part and the modulus of the loop. Figure shows that the simple zeroing imaginary part can not be excluded from the bending schedule. As well as for the case of a local loop, the integral Π (p^2) has a branch point at $p^2 = 4m^2$, since in this case the value of the exponent z_0 becomes zero in y = 1/2, and the integral over the variable t begin to diverge at infinity. You can see that the introduction of the infrared cutoff for a loop with a nonlocal form factor, the curve is smooth and does not contain curves.

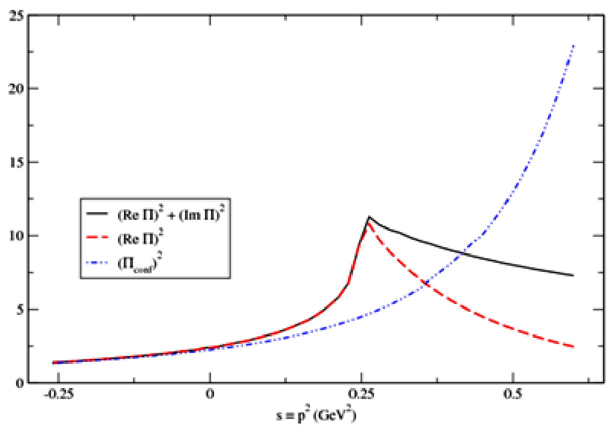

Figure 2 – The square of real part, modulus of loop and loop with infrared confinement

Conclusion

As a result of studies of the role of confinement in a covariant quark model the following results are obtained: With the help of exact analytical calculations of the local quark loop with the scalar quarks one demonstrated by the behavior of its real and imaginary parts. In the more complicated case, where the quark loop has a nonlocal vertex form factor, one using the numerical calculations also obtained the behavior of both the real and imaginary parts. Performed calculations of quark loop with nonlocal vertex functions and taking into account the quark confinement that is use din covariant quark model. It is shown that in the latter case there is no the imaginary part of the diagram, as well as-bending ("cusp") corresponds to the threshold of quark production.

Reference

1. Weinberg S. Elementary particle theory of composite particles. // Phys. Rev. 1963. – Vol. 130. – P. 776-783.

2. Salam A. Lagrangian Theory of Composite Particles //NuovoCim. – 1963. – Vol.25.– P.224.

3. Dineykhan M., Ivanov M.A., Saidullaeva G.G. Exotic states and rare *Bs* decays in the covariant quark model// Physics of Particles and Nuclei. – 2012. – Vol.43.– P.749-782.

4. Branz T., Faessler A., Gutsche T., Ivanov M.A., Körner J.G., Lyubovitskij V.E. Relativistic constituent quark model with infrared confinement // Phys. Rev. D – 2010. – Vol.81. – P.034010.