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Solar-assisted auto-cascade heat pump for water heating in the continental climates

Abstract. Heat pumps are widely recognized as energy-efficient technologies for domestic hot water production. However, conventional single-stage vapor compression heat pumps utilizing ambient air as the heat source are limited in delivering outlet water temperatures above 323 K under low ambient conditions. This limitation restricts their applicability in continental climates characterized by large diurnal and seasonal temperature variations. Two-stage cascade systems can achieve higher outlet temperatures exceeding 343 K, but they require two compressors, resulting in increased energy consumption and higher capital costs. To overcome these drawbacks, the present study proposes an auto-cascade compression heat pump system employing an environmentally friendly binary zeotropic refrigerant mixture to achieve water outlet temperatures above 343 K with improved efficiency and reduced system complexity. In addition, solar collectors are integrated to enhance low-grade heat extraction from the environment. A numerical simulation of the proposed auto-cascade system was conducted for binary zeotropic refrigerant mixtures including R32/R134a, R32/R1234yf, R32/R1234ze, and R32/R245fa within an ambient temperature range of 223–273 K. The results show that the coefficients of performance (COP) for R32/R134a, R32/R1234yf, and R32/R1234ze mixtures vary between 2.72 and 2.75, while that of R32/R600a reaches 2.55. Based on the comparative analysis, the R32/R134a mixture demonstrated the best performance and is recommended as a promising working fluid for auto-cascade heat pump systems designed for water heating applications in continental climate regions.
Keywords: Auto-cascade Heat Pump, Zeotropic Mixture, Continental Climate, Water Heating, COP.

Introduction

Space and water heating in continental climates rely predominantly on fossil fuels such as coal, natural gas, and diesel. These systems, which include both centralized district heating and decentralized domestic boilers, are major contributors to greenhouse gas emissions and local air pollution during the heating season. In many regions of Central Asia and the Commonwealth of Independent States (CIS), including Kazakhstan, Russia, and Uzbekistan, the use of coal-fired power plants and residential furnaces remains widespread due to low fuel prices and underdeveloped renewable infrastructure. However, these traditional heating systems exhibit low overall efficiency and impose substantial environmental impacts, particularly in densely populated urban areas and unregulated rural zones [1-2].

Heat pump technologies represent an efficient and sustainable alternative for residential heating and hot-water production, as they can utilize renewable and low-grade heat sources such as ambient air, solar radiation, and the ground. Among various configurations, solar-assisted heat pumps (SAHPs) have attracted increasing attention as they combine solar thermal energy with vapor compression cycles to improve the coefficient of performance (COP) and reduce electricity consumption. Comprehensive reviews by Mohanraj et al. [1,2] summarized two decades of advancements in SAHP systems, including modeling approaches, performance enhancement techniques, and environmentally friendly refrigerants. These studies emphasized that ground-source heat pumps (GSHPs) are highly suitable for cold climates; however, their adoption is constrained by high drilling costs and complex installation. Conversely, air-source heat pumps (ASHPs) are cost-effective and easy to install but exhibit poor performance at low ambient temperatures, as conventional single-stage vapor compression systems are generally unable to deliver outlet water temperatures above 323 K under cold conditions.

To address these limitations, Yerdesh et al. [3] developed direct and indirect expansion two-stage cascade heat pump systems integrated with solar energy for improved performance in Kazakhstan's continental climate. Although cascade configurations can achieve outlet temperatures above 343 K, they require two compressors, which increase system complexity, power consumption, and capital costs. Therefore, it is essential to develop a system capable of achieving high-temperature output with a single compressor while maintaining low operating and investment costs.

One promising solution is the auto-cascade compression cycle [4-5], originally developed for cryogenic refrigeration applications. This cycle employs zeotropic refrigerant mixtures to achieve multi-temperature evaporation and condensation within a single loop, thereby eliminating the need for multiple compressors [6–8]. Recent advancements in auto-cascade refrigeration systems have demonstrated significant improvements in thermodynamic efficiency through techniques such as ejector expansion and phase separation enhancement [9–12]. For instance, Liu et al. [12] reported that an ejector-expansion auto-cascade refrigeration system using an R290/R170 mixture achieved 80% higher COP and 78.5% greater volumetric capacity compared with a conventional cycle.

Although the auto-cascade concept has been extensively investigated for refrigeration applications, relatively few studies have explored its potential for heating and water-heating systems [13–16]. Zhao et al. [13] analyzed the performance of an auto-cascade heat pump (ACHP) at an ambient temperature of 263 K and a heating temperature of 323 K, identifying R143a/R600 (0.8/0.2) as the optimal mixture with a COP of 2.15. Lv et al. [14] proposed a solar-assisted auto-cascade heat pump (SAHP) using the zeotropic mixture R32/R290 for small-scale water heating, reporting COP and volumetric heating capacity improvements of 9.85% and 9.68%, respectively. Yu et al. [15] studied CO_2–propane auto-cascade systems for electric vehicle heating, showing superior performance compared with conventional transcritical CO_2 cycles at subzero temperatures. Fan et al. [16] further enhanced the concept by introducing an ejector-assisted internal auto-cascade heat pump using R32/R290, achieving up to 19% improvement in COP for district heating in cold regions.

Building upon these prior efforts, the present study proposes a solar-assisted auto-cascade heat pump (SAAHP) system utilizing binary zeotropic refrigerant mixtures and a direct-expansion solar collector [17, 18, 19]. This configuration eliminates the need for an intermediate coupling fluid loop and enhances low-grade heat extraction from both solar and ambient sources. The proposed system aims to deliver hot water above 343 K suitable for domestic use in continental climates while reducing compressor power consumption and overall system cost. Numerical simulations are performed to evaluate the thermal performance of the system using various environmentally friendly refrigerant mixtures (R32/R134a, R32/R1234yf, R32/R1234ze, R32/R245fa, and R32/R600a), focusing on COP variation with ambient temperature and condensation conditions. The modeling methodology in this manuscript builds on the experimentally validated framework presented by Yerdesh et al. [20], ensuring the reliability of the thermodynamic predictions.

System Description

Figures 1 and 2 illustrate the configuration of the proposed solar-assisted auto-cascade heat pump (SAAHP) system and its corresponding pressure–enthalpy (P–h) diagram. The working principle of the cycle can be described as follows. A low-pressure vapor mixture of refrigerants (state 1) is first compressed adiabatically by a single compressor to the condensation pressure (state 2). The high-pressure vapor mixture then passes through the condenser (state 3), where it rejects heat to a domestic water stream and condenses into a liquid phase. The condensed liquid mixture is subsequently throttled through an expansion device to an intermediate pressure (state 4), partially flashing into a two-phase state. This two-phase mixture enters a direct-expansion (DX) solar collector [21], where it absorbs low-grade solar thermal energy and continues to evaporate (state 5).

The vapor–liquid mixture leaving the solar collector enters a phase separator (state 6), where the more volatile (low-boiling-point) components are separated from the less volatile (high-boiling-point) liquid. The liquid fraction is expanded further to a lower pressure and temperature (state 7) and then warmed recuperatively by condensing the vaporized low-boiling-point fraction within the internal heat exchanger (state 8). The low-boiling-point vapor subsequently condenses into a subcooled mixture (state 9–10), which passes through a secondary recuperative subcooler (state 11) before being expanded to sub-ambient pressure (state 12). At this stage, it evaporates by extracting additional heat from the ambient air (state 13).

Finally, the vapor streams from the solar and ambient evaporators merge and return to the compressor inlet (state 14), thereby completing the cycle. Consequently, the proposed SAAHP simultaneously utilizes solar irradiation (process 4 → 5) and ambient air (process 12 → 13) as renewable heat sources to deliver high-temperature water for residential heating applications while maintaining a single-compressor configuration and improved thermodynamic efficiency.

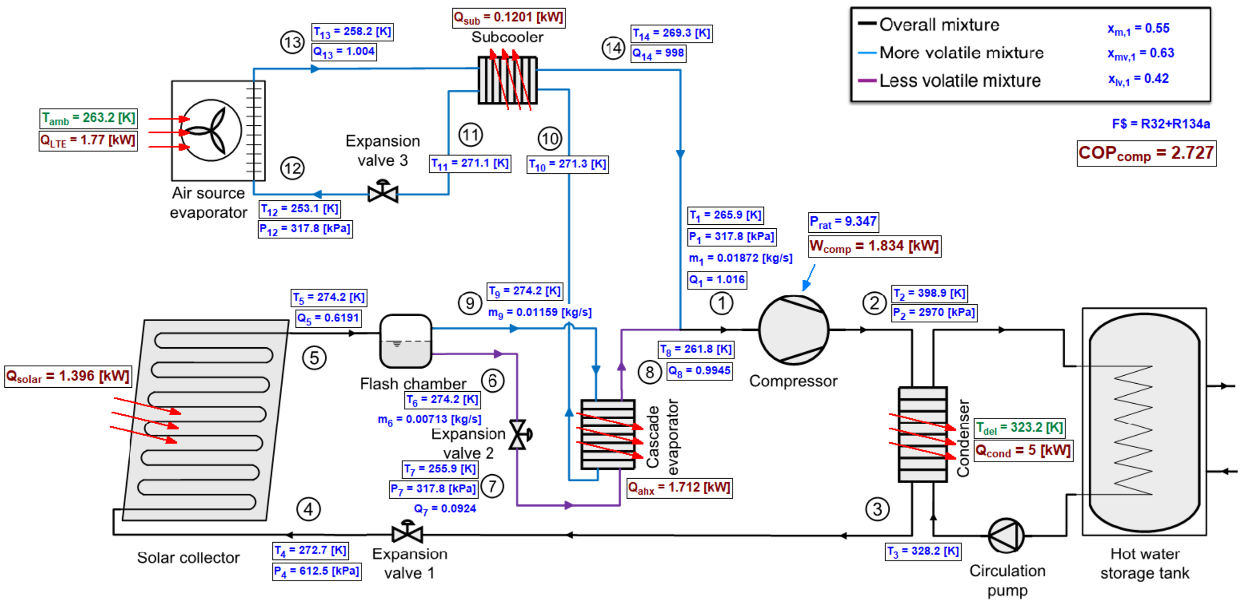

Figure 1 – Schematic diagram of the proposed solar-assisted auto-cascade heat pump (SAAHP) system configuration

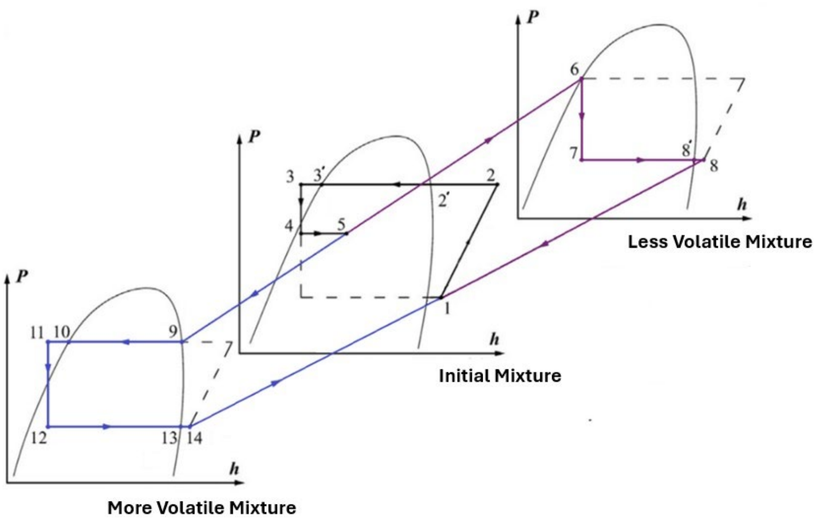

Figure 2 – Pressure–enthalpy (P–h) diagram illustrating the thermodynamic processes of the proposed solar-assisted auto-cascade heat pump (SAAHP) cycle

The operating principle of the solar-assisted auto-cascade heat pump (SAAHP) is illustrated in the pressure–enthalpy (P–h) diagram presented in Figure 2. The thermodynamic processes of the cycle can be described as follows:

- 1–2 (Adiabatic compression): The binary zeotropic refrigerant mixture is compressed adiabatically from the initial evaporation pressure (P_{low}) to the condensation pressure by a single-stage compressor.
- 2′–3′ and 9–10 (Isothermal and isobaric condensation): The refrigerant mixture undergoes condensation at constant pressure and temperature in the main condenser and the internal auto-cascade heat exchanger, respectively. The corresponding isobaric segments are denoted as 2–3 and 9–11.
- 3–4 (Isenthalpic expansion): The condensed refrigerant mixture is throttled through an expansion valve from the condensation pressure (P_{high}) to an intermediate pressure (P_{med}), resulting in partial flashing into a two-phase mixture.
- 6–7 and 11–12 (Isenthalpic throttling): The less volatile and more volatile fractions of the refrigerant mixture are each expanded through separate throttling valves from the intermediate pressure (P_{med}) to the low-pressure (P_{low}) level of the system.
- 7–8′ and 12–13 (Isothermal and isobaric evaporation): The refrigerant mixtures evaporate at constant temperature and pressure within the internal auto-cascade heat exchanger and the ambient air evaporator, respectively. The corresponding isobaric sections are represented by 7–8 and 12–14.

Thermal energy is absorbed from two renewable sources [22]: solar radiation via the direct-expansion solar collector (process 4–5) and ambient air via the air-source evaporator (process 12–13). Superheating of the vapor phase occurs in the auto-cascade heat exchanger for the less volatile components (8′–8) and in the ambient air heat exchanger for the more volatile components (13–14), ensuring stable operation and efficient energy recovery throughout the cycle.

Mathematical Model

To accurately predict the performance of the proposed solar-assisted auto-cascade heat pump (SAAHP), a detailed thermodynamic model was developed. Since the system employs a binary zeotropic refrigerant mixture, determining the appropriate concentration of the more volatile and less volatile components is essential. Initially, a parametric evaluation of the refrigerant composition was performed to identify the mixture ratios that provide the optimal balance between heating capacity and desired temperature lift [23]. Using these results, a comprehensive energy model was constructed to calculate the heating capacities of the solar collector and air-source evaporator, the heat delivered by the condenser, and the compressor power input.

The modeling framework is based on the first law of thermodynamics, with the following simplifying assumptions:

(1) The entire system operates under steady-state conditions.

(2) Changes in potential, kinetic, and chemical energies are negligible.

(3) Pressure drops in the suction and discharge pipelines of the compressor are ignored.

(4) The isentropic efficiency of the compressor is taken as 0.8.

(5) Evaporation and condensation processes occur at constant pressure.

(6) Throttling in the expansion valves is assumed to be isenthalpic.

(7) Superheating at the evaporator outlet and subcooling at the condenser outlet are both assumed to be 5 K [3].

The high-side pressure of the cycle, corresponding to the condenser outlet, is evaluated under the assumption that the refrigerant mixture is completely condensed into a liquid phase. It is defined by:

$$P_{high} = f(T_{cond} + Sub_{cool}; x_3; w_{o,1}) \quad (1)$$

$$T_{cond} = T_{del} + CAT \quad (2)$$

here, T_{cond} is the condenser outlet temperature (state point 3), Sub_{cool} is the subcooling temperature (Sub_{cool}= 5 K), x_3 denotes the vapor quality (for a saturated liquid, x_3= 0), and $w_{o,1}$ represents the mass fraction of the more volatile refrigerant.

The low-side pressure is determined assuming that the more volatile component is fully vaporized at the outlet of the air-source evaporator:

$$P_{low} = P(T_{LTE} - Sup_{LTE}; x_{13}; w_{mv,1}) \quad (3)$$

$$T_{LTE} = T_{amb} - CAT \quad (4)$$

here, T_{LTE} represents the temperature of the more volatile components at the outlet of the air-source evaporator (state point 13). Sup_{LTE} denotes the superheating temperature (Sup_{LTE}= 5 K), and x_{13} is the vapor quality (for a saturated vapor, x_{13}= 1). The term $w_{mv,1}$ refers to the mass fraction of the more volatile refrigerant in the vapor phase of the mixture.

The compressor pressure ratio is defined as:

$$P_{ratio} = \frac{P_{high}}{P_{low}} \quad (5)$$

The intermediate pressure is determined using two independent criteria. First, it must correspond to the temperature of the more volatile refrigerant mixture at the outlet of the separator (state point 9):

$$P_{med} = P(T_{sol}; x_9; w_{mv,1}) \quad (6)$$

Conversely, the intermediate pressure must also satisfy the temperature condition of the less volatile liquid refrigerant mixture at the outlet of the separator (state point 6):

$$P_{med} = P(T_{sol}; x_6; w_{lv,1}) \quad (7)$$

here, T_{sol} denotes the temperature of the initial refrigerant mixture at the outlet of the solar collector, and $w_{lv,1}$ represents the mass fraction of the more volatile refrigerant contained within the less volatile liquid mixture.

The relationship between the overall refrigerant mixture and its more and less volatile components can be expressed by the following ratio:

$$x_{sol} \cdot w_{mv,1} + (1 - x_{sol}) \cdot w_{lv,1} = w_{o,1} \quad (8)$$

Here, x_{sol} denotes the vapor quality of the refrigerant mixture after passing through the solar collector, corresponding to a state of partial evaporation (approximately half-evaporated).

The optimal concentration of refrigerants in the zeotropic mixture is determined based on the calculated values of P_{high}, P_{low} and P_{ratio}. Using these parameters, a complete thermodynamic model of the heat pump is established to evaluate the overall system performance and efficiency.

The mathematical formulation of the SAAHP system (as summarized in Table 1) is based on determining the thermodynamic properties of the refrigerant mixture at each characteristic state point within the cycle.

At the compressor inlet (state point 1), the enthalpy of the returning vapor mixture is assumed to be equal to that of the initial mixture, in accordance with the principle of energy conservation:

$$\dot{m}_0 h_1 = \dot{m}_{mv} h_{14} + \dot{m}_{lv} h_8 \quad (9)$$

For the SAAHP system, the coefficient of performance (COP) is evaluated using the following expression:

$$COP = \frac{Q_{cond}}{W_{comp}} \quad (10)$$

Performance Calculations

Preliminary computational results were obtained using the developed mathematical model implement-ted in the Engineering Equation Solver (EES) software [24]. The simulations of the SAAHP system, performed according to Eqs. (1)–(10) and the component models summarized in Table 1, utilized the following input parameters: ambient air temperature T_{amb}, required water temperature in the heating circuit T_{del}, and the corresponding heating capacity (Q_{cond}) of the SAAHP for various binary refrigerant mixtures.

In the auto-cascade system, a binary zeotropic refrigerant mixture serves as the working fluid. The components of the mixture are distinguished by their differing boiling points, which enable phase separation and heat exchange within the cycle. For the SAAHP performance analysis, several binary mixtures were investigated, with R32 selected as the low-boiling-point component. The secondary (high-boiling-point) refrigerants considered in combination with R32 included R600a, R134a, R1234yf, R1234ze, and R245fa.

Depending on the temperature range, that is, on the specified ambient temperature T_{amb} and the desired heat delivery temperature T_{del}, the corresponding values of P_{high}, P_{low} and the pressure ratio P_{ratio} were determined using Eqs. (1)–(5). Assumptions were made regarding the overall mass fraction of the refrigerants within the zeotropic mixture. By solving Eqs. (1)–(5), the operating pressures of the system and the compressor pressure ratio were obtained for binary zeotropic mixtures based on R32. In the flash chamber (phase separator), which separates the liquid and vapor phases of the mixture, the mass flow rates of the liquid and vapor fractions, $w_{mv,1}$ and $w_{lv,1}$, were specified accordingly.

Table 1 – Thermodynamic models of the main components of the SAAHP system

Key points	Based on assumptions	Models based on the first law of thermodynamics	Based on refrigerant properties
1	$P_1 = P_{low}$	$h_1 = f(P_1; T_1; w_{0,1})$	$s_1 = s(P_1; T_1; w_{o1})$ $x_1 = x(P_1; T_1; w_{o,1})$
2	$P_2 = P_{high}$	$W_{comp} = \dot{m}_0(h_2 - h_1)$ $W_{comp} = \dot{m}_0(h_{s,2} - h_1)/\eta_{comp}$	$h_{s,2} = h(P_2; s_1; w_{o,1})$ $T_2 = T(P_2; h_2; w_{o,1})$ $s_2 = s(P_2; h_2; w_{o,1})$
3	$P_3 = P_2$ $T_3 = T_{cond}$	$Q_{cond} = \dot{m}_0(h_2 - h_3)$	$h_3 = h(P_3; T_3; w_{o,1})$ $s_3 = s(P_3; T_3; w_{o,1})$
4	$P_4 = P_{med}$ $h_4 = h_3$		$T_4 = T(P_4; h_4; w_{o,1})$ $s_4 = s(P_4; h_4; w_{o,1})$
5	$P_5 = P_4$ $T_5 = T_{sol}$	$Q_{solar} = \dot{m}_0(h_5 - h_4)$	$s_5 = s(P_5; h_5; w_{o,1})$ $x_5 = x(P_5; T_5; w_{o,1})$
6	$P_6 = P_5$ $T_6 = T_5$	$\dot{m}_{lv} = (1 - x_5) \cdot \dot{m}_0$	$h_6 = h(P_6; T_6; w_{lv,1})$ $s_6 = s(P_6; T_6; w_{lv,1})$
7	$P_7 = P_{low}$ $h_7 = h_6$		$T_7 = T(P_7; h_7; w_{lv,1})$ $s_7 = s(P_7; h_7; w_{lv,1})$ $x_7 = x(P_7; h_7; w_{lv,1})$
8	$P_8 = P_7$	$Q_{AHX} = \dot{m}_{lv}(h_8 - h_7)$	$T_8 = T(P_8; h_8; w_{lv,1})$ $s_8 = s(P_8; h_8; w_{lv,1})$ $x_8 = x(P_8; h_8; w_{lv,1})$
9	$P_9 = P_5$ $T_9 = T_5$	$\dot{m}_{mv} = x_5 \cdot \dot{m}_0$	$h_9 = h(P_9; T_9; w_{mv,1})$ $s_9 = s(P_9; T_9; w_{mv,1})$
10	$P_{10} = P_9$	$Q_{AHX} = \dot{m}_{mv}(h_9 - h_{10})$	$T_{10} = T(P_{10}; h_{10}; w_{mv,1})$ $s_{10} = s(P_{10}; h_{10}; w_{mv,1})$
11	$P_{11} = P_{10}$	$Q_{sub} = \dot{m}_{mv}(h_{10} - h_{11})$	$T_{11} = T(P_{11}; h_{11}; w_{mv,1})$ $s_{11} = s(P_{11}; h_{11}; w_{mv,1})$
12	$P_{12} = P_{low}$ $h_{12} = h_{11}$		$T_{12} = T(P_{12}; h_{12}; w_{mv,1})$ $s_{12} = s(P_{12}; h_{12}; w_{mv,1})$ $x_{12} = x(P_{12}; h_{12}; w_{mv,1})$
13	$P_{13} = P_{12}$ $T_{13} = T_{LTE}$	$Q_{LTE} = \dot{m}_{mv}(h_{13} - h_{12})$	$h_{13} = h(P_{13}; T_{13}; w_{mv,1})$ $s_{13} = s(P_{13}; T_{13}; w_{mv,1})$ $x_{13} = x(P_{13}; h_{13}; w_{mv,1})$
14	$P_{14} = P_{13}$	$Q_{sub} = \dot{m}_{mv}(h_{14} - h_{13})$	$T_{14} = T(P_{14}; h_{14}; w_{mv,1})$ $s_{14} = s(P_{14}; h_{14}; w_{mv,1})$ $x_{14} = x(P_{14}; h_{14}; w_{mv,1})$

Figure 3 presents the calculated results for an ambient temperature $T_{amb} = 263\,K$ and a heat delivery temperature $T_{del} = 313\,K$, illustrating the variation of the coefficient of performance (COP) and compression ratio for different binary refrigerant mixtures.

According to the simulation results, the mixtures R32/R134a, R32/R1234yf, and R32/R1234ze exhibit the highest coefficients of performance, with COP values of approximately 3.19. The system performance using these mixtures is 7–10% higher compared to that obtained with R32/R600a and R32/R245fa.

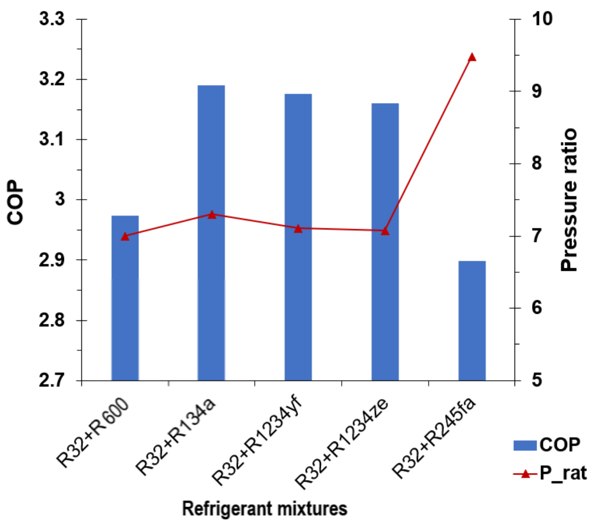

Figure 3 – Variation of the coefficient of performance (COP) and pressure ratio for different binary refrigerant mixtures

The vapor quality x_{sol} and temperature T_{sol} were defined within the direct-expansion (DX) type solar collector. Based on the calculated data for all refrigerant mixtures, the vapor quality, representing the degree of evaporation of the refrigerant mixture after the solar collector, was found to be in the range of $x_{sol} = 0.45 \div 0.50$. To achieve approximately half evaporation in the solar collector, the operating temperature for the investigated mixtures should reach values in the range of 278-283 K.

Figures 4 and 5 illustrate the variation of the coefficient of performance (COP) and pressure ratio as functions of the assumed vapor quality and operating temperature within the solar collector. The calculations were performed considering only daytime operating conditions, during which the system has access to available solar radiation.

According to Figure 3, the maximum coefficient of performance (COP) of approximately 2.75 is achieved at a vapor quality of about 0.45. As the vapor quality increases, the COP decreases due to a corresponding rise in the pressure ratio, which increases the compressor load. Therefore, subsequent simulations were performed under the assumption of partial (approximately 50%) evaporation after the solar collector. Furthermore, variations in the solar collector temperature within the investigated range were found to have only a minor influence on the COP, as illustrated in Figure 5.

Figures 6 and 7 present the variation of the coefficient of performance (COP) with respect to the outdoor air temperature and the condensing temperature, respectively. Specifically, Figure 7 illustrates the dependence of both COP and pressure ratio on the condensing temperature for the R32/R134a refrigerant mixture.

According to Figure 6, an increase of 1 K in the outdoor air temperature results in an 8–10% rise in the coefficient of performance (COP). As shown, the R32/R134a refrigerant mixture exhibits the highest COP within the investigated temperature range. However, R134a possesses a relatively high global warming potential (GWP) of 1430, and its use is scheduled for phase-out under the 2016 Paris Agreement. Additionally, R32 has a moderate GWP of approximately 650 [3]. For the R32/R134a mixture, the COP at an outdoor air temperature of 255 K and a heat delivery temperature of 313 K is approximately 2.8.

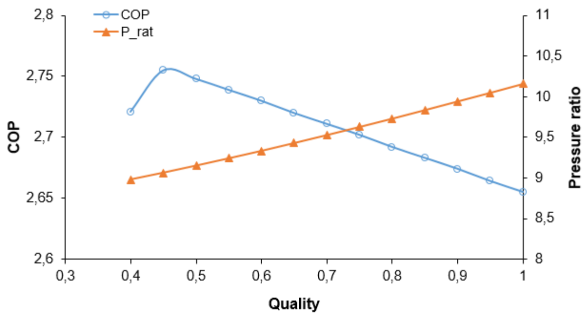

Figure 4 – Variation of the coefficient of performance (COP) and pressure ratio as a function of vapor quality in the solar collector

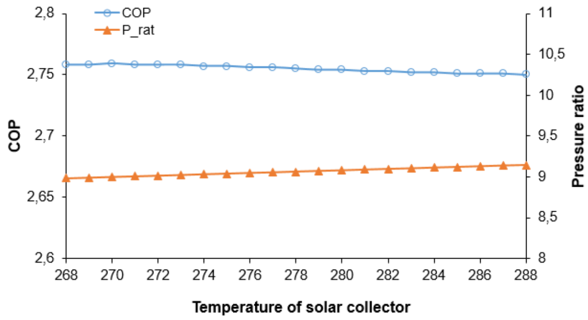

Figure 5 – Variation of the coefficient of performance (COP) and pressure ratio as a function of operating temperature in the solar collector.

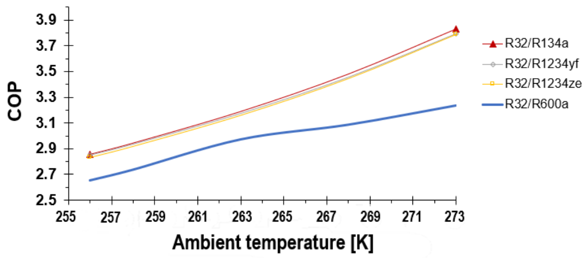

Figure 6 – Variation of the coefficient of performance (COP) as a function of outdoor air temperature.

Figure 7 illustrates the relationship between the coefficient of performance (COP) and pressure ratio as functions of the condensing temperature for the R32/R134a refrigerant mixture. In this analysis, the outdoor air temperature is maintained at a constant value of 263 K. As shown, a reduction in the condensing temperature leads to a lower compression ratio and a corresponding increase in COP. For instance, at a condensing temperature of 303 K, the COP reaches approximately 4.0.

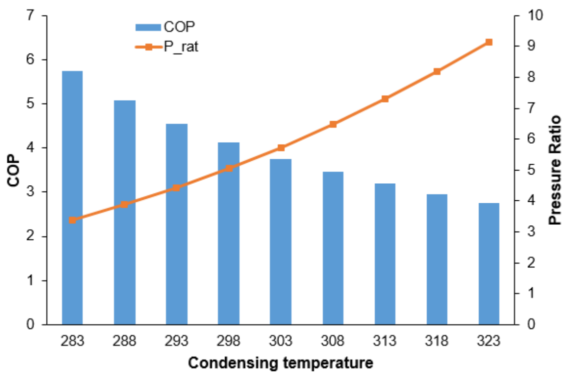

Figure 7 – Variation of the coefficient of performance (COP) and pressure ratio as functions of condensing temperature.

The direct experimental testing of the proposed solar-assisted auto-cascade system has not yet been performed, and the modeling approach has been previously validated against full-scale experiments in Yerdesh et al. [20].

Conclusions

The thermal performance of a solar-assisted auto-cascade heat pump (SAAHP) designed for domestic water heating applications was numerically investigated under varying ambient and condenser operating temperatures. The main findings can be summarized as follows:

- A detailed mathematical model based on the first law of thermodynamics was developed to predict the thermodynamic behavior and efficiency of the SAAHP system.

- The simulations were performed considering specific assumptions regarding vapor quality and temperature at the outlet of the solar collector, as well as the distribution of the more volatile and less volatile components within the phase separator.

- Within the investigated temperature range– ambient air temperature of 255 K and heat delivery temperature of 313 K, the coefficient of performance (COP) varied between 2.62 and 2.82 for all tested refrigerant mixtures. Among the mixtures analyzed, R32/R134a and R32/R1234yf demonstrated the highest overall performance.

- The model assumed system operation only during daytime when solar radiation is available. Therefore, for continuous year-round operation, the SAAHP should be integrated with auxiliary heat sources such as gas or electric boilers.

The future work will involve experimental construction and validation of the proposed auto-cascade system to further strengthen the findings.

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