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Exploring the Impact of Anisotropy Parameters on Stellar Structure

Abstract. In this study, we examine how variations in local pressure anisotropy affect the internal structure and equilibrium of white dwarfs. A generalized anisotropy model is developed, defined by three parameters: the amplitude coefficient α_0 and the shape exponents l and k. This formulation ensures that the anisotropic pressure continuously vanishes at both the center and surface of the star while reaching a single peak in the intermediate region. By applying appropriate boundary and regularity conditions, the model allows us to determine physically stable parameter domains consistent with realistic stellar configurations. Our analysis shows that even a small degree of anisotropy can have a measurable effect on the mass-radius relation and overall compactness of white dwarfs, which may help explain the origin of super-Chandrasekhar systems observed in astrophysical data. This modeling approach provides a clear and flexible way to describe compact stars with anisotropic pressures and can also be applied to neutron and quark stars.
Keywords: compact stars, white dwarfs, anisotropic pressure, stability analysis, generalized anisotropic factor.

Introduction

White dwarfs, neutron stars, and black holes serve as natural laboratories for probing general relativity in strong gravitational fields. The equilibrium of such dense objects arises from the interplay between the curvature of spacetime and the distribution of matter, as expressed through Einstein's field equations [1–3]. In the case of white dwarfs, the internal balance between gravity and pressure is usually represented by the Tolman–Oppenheimer–Volkoff (TOV) equations, which describe hydrostatic equilibrium under the assumption of isotropic pressure, meaning that the radial and tangential components are equal [4,5].

In realistic astrophysical conditions, strong magnetic fields, rapid rotation, and phase transitions can break isotropy, producing directional pressure differences inside compact stars[6–10]. When these effects become significant, the radial pressure p_r no longer coincides with the tangential pressure p_t, and their difference changes both the internal density profile and global parameters such as total mass, radius, and compactness.

Recent observations indicate that compact stars may exhibit measurable pressure anisotropy. The

supernovae and highly magnetized white dwarfs points to additional internal stresses that may serve as extra pressure support beyond the isotropic limit [13–15]. Theoretical results further suggest that anisotropic stresses can increase the maximum stable mass and shift the boundaries of stability, implying that anisotropy plays a crucial role in the structural and evolutionary properties of compact stars. Therefore, incorporating anisotropy into stellar models allows for a more realistic interpretation of precision astrophysical observations.

The theoretical treatment of anisotropy in self-gravitating systems has a long history. Bowers and Liang proposed a linear dependence of anisotropy on density and radius[6]. Later developments by Dev and Gleiser, Herrera and Santos, and other authors introduced more general formulations, including nonlinear or gradient-dependent terms that relate the anisotropy to variations in the gravitational potential and energy density [7,16,17].

Our previous investigation [18] demonstrated that even a moderate level of anisotropy can significantly modify the equilibrium configuration and shift the mass–radius relation toward more compact states. These results motivated us to develop of describing a broader range of anisotropic behaviors.

In this study, we extend earlier models by introducing a generalized anisotropy function defined by three parameters: the strength coefficient α_0, and two shape parameters l and k. This functional form ensures that anisotropy smoothly vanishes at the stellar center and surface while exhibiting a single extremum within the interior region, consistent with boundary and regularity requirements. The proposed parameterization unifies several existing models as limiting cases and allows a systematic study of how anisotropy affects stellar equilibrium across different regimes.

Beyond static balance, anisotropy may influence dynamic and observational characteristics of compact objects, including oscillation frequencies, collapse processes, gravitational redshift, and emission properties [19–22]. The purpose of this study is to explore how changes in the anisotropy parameters influence the internal pressure profile of compact stars and to identify regions of parameter space that correspond to stable equilibrium states. The developed approach serves as a groundwork for future analyses of anisotropic behavior in various dense astrophysical objects, including neutron and quark stars.

Theoretical framework

To examine how pressure anisotropy affects stellar equilibrium, we consider a static, spherically symmetric stellar configuration described by the metric:

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - \\ -r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \quad (1)$$

In this metric, $\nu(r)$ and $\lambda(r)$ are gravitational potential functions determined by Einstein's field equations. The energy–momentum tensor for an anisotropic fluid is given by:

$$T^{\alpha}_{\beta} = diag\left(\rho, -p_r, -p_r, -p_t\right), \quad (2)$$

In this tensor, ρ represents the energy density, p_r the radial pressure, and p_t the tangential pressure. The degree of anisotropy is characterized by the difference:

$$\Delta(r) = p_t(r) - p_r(r), \quad (3)$$

where $\Delta(r)$ is referred to as the anisotropy factor, A positive $\Delta > 0$ corresponds to additional tangential pressure support, whereas a negative $\Delta < 0$ indicates that radial pressure dominates.

By substituting the anisotropic energy–momentum tensor into Einstein's field equations, we obtain the Tolman–Oppenheimer–Volkoff (TOV) equations with an extra anisotropy term [4,5], given as Equations (4) and (5).

$$\frac{dm}{dr} = 4\pi r^2 \rho \quad (4)$$

$$\frac{dp_r}{dr} = -\frac{(\rho + p_r)(m + 4\pi r^3 p_r)}{r(r-2m)} + 2\frac{\Delta}{r} \quad (5)$$

These equations govern the hydrostatic equilibrium of an anisotropic fluid sphere. In the limiting case $\Delta = 0$, where $p_t = p_r$, the anisotropic contribution vanishes and the standard isotropic TOV solution is recovered.

Pressure Anisotropy in Compact Stars

When modeling the interior of a compact star, the usual assumption is that pressure is isotropic, meaning it is the same in all directions. However, under many realistic physical conditions this assumption breaks down. In such cases, the radial pressure p_r no longer equals the tangential pressure p_t, and the configuration develops pressure anisotropy. The anisotropy factor Δ defined above quantifies the magnitude of this pressure difference.

Several physical mechanisms can give rise to pressure anisotropy, including:

Strong magnetic fields, which introduce directional pressure gradients [8].

Rapid rotation, which leads to equatorial flattening and an uneven pressure distribution [6].

Phase transitions in dense matter, for example quark deconfinement or crystallization, which alter the internal pressure balance [12].

Local microphysical interactions in self-bound or charged fluids, which produce internal anisotropic stresses [7, 16].

Representative Models of Anisotropy

A number of phenomenological models for the anisotropy factor Δ have been proposed in the literature. For instance:

- Bowers & Liang (1974): This model assumes that Δ grows linearly with the local density and the radial coordinate [6] (see Equation (7)). In their linear formulation, λ is a constant and R is the total stellar radius.

$$\Delta(r) = \lambda\rho(r)\left(1-\frac{r}{R}\right), \tag{6}$$

- Dev & Gleiser (2002): These authors proposed a power-law form for the anisotropy factor [7], given by Equation (8). The power-law behavior yields only a mild anisotropic effect near the center of the star, but allows much stronger deviations from isotropy toward the stellar surface.

$$\Delta(r) = \alpha r^2\left(\rho + p_r\right) \tag{7}$$

- Herrera & Santos (1997): Herrera and Santos outlined general criteria for anisotropy and presented models in which Δ depends explicitly on the pressure gradient and the gravitational potential [8]. This approach effectively describes anisotropy arising from underlying thermodynamic forces within the star.

Each of these models offers certain advantages but also has notable limitations in flexibility. In particular, it can be challenging for any single model to satisfy observational constraints while simultaneously maintaining physically reasonable behavior at the star's center and surface.

Behavior of the Generalized Anisotropic Factor

Dimensionless Transformation and Parameter Scaling

In this work, we propose a generalized form of the anisotropy factor that introduces three tunable parameters [18]:

$$\Delta(r) = \alpha r^l\left(1-\left(\frac{r}{R}\right)^k\right) \tag{8}$$

where α is a dimensional constant that controls the strength of anisotropy, and *l*, *k* are dimensionless shape parameters. The function vanishes at the center (r = 0) and at the surface (r = R), and reaches a maximum or minimum at some intermediate radius.

To simplify numerical integration, we introduce dimensionless variables:

$$x = \frac{r}{b},\; x_f = \frac{R}{b}, \tag{9}$$

where b is a characteristic length scale that depends on the central density and the equation of state. By equating the dimensional and dimensionless expressions, we obtain

$$\tilde{\Delta}(x) = \alpha_0\frac{c^4}{Gb^{2+l}}x^l\left(1-\left(\frac{x}{x_f}\right)^k\right) \tag{10}$$

which leads to the scaling relation

$$\alpha = \alpha_0\frac{c^4}{Gb^{2+l}}, \tag{11}$$

here, b plays an essential role in connecting the model's dimensionless formulation to physical quantities. Its value is computed numerically and reflects the influence of both central density and microphysical assumptions.

The characteristic pressure in relativistic stellar models is typically of the order

$$p_c \sim \frac{c^4}{Gb^2}. \tag{12}$$

To ensure that the anisotropic pressure remains smaller than the dominant isotropic term, we typically constrain α_0 within the interval $-1<\alpha_0<1$. This condition helps maintain hydrostatic equilibrium and avoids instabilities that could arise if anisotropic contributions were too large near the core.

Boundary conditions

We next analyze the mathematical structure of the generalized anisotropic factor given in dimensionless form by

$$\tilde{\Delta}(x) = \alpha_0 x^l \left(1 - x^k\right) \qquad (13)$$

where x = r/R is the normalized radial coordinate, and *l, k* are shape-controlling parameter, for numerical calculations we take $R = 1$.

The function of anisotropic factor is designed to satisfy three physical conditions:

- Vanishing at the center of the star: $\tilde{\Delta}(x) = 0$
- Vanishing at the surface of the star: $\tilde{\Delta}(1) = 0$

Regularity conditions

To ensure $\tilde{\Delta}(x) = 0$, the x^l must approach zero as x ⟶ 0. This is satisfied when

$$l > 0 \qquad (14)$$

At the surface $x = 1$, the term $1 - x^k$ vanishes for any nonzero k, so we have

$$\tilde{\Delta}(1) = \alpha_0 1^l \left(1 - 1^k\right) = 0 \qquad (15)$$

which holds generally for all real $k \neq 0$.

Numerical Analysis of Parameters *l, k* and α_0

This section presents a systematic numerical analysis of the generalized anisotropic factor defined in Eq. (13), focusing on its behavior under various combinations of the parameters *l, k*, and the strength coefficient α_0. The function is designed to vanish at the center and the surface of the star, and to reach an extremum at an intermediate radius. Our aim is to classify parameter sets into physically viable, edge-case, or irregular regimes based on the regularity and structure of the anisotropy profile.

$$For \ -1 \leq \alpha_0 < 1$$

To investigate the influence of the shape parameters l and k, we computed the anisotropy factor $\tilde{\Delta}(x)$ for a range of values with $\alpha_0 \in [-1, 1]$, which ensures that the anisotropic pressure remains comparable to the isotropic pressure. Each figure in this subsection corresponds to a fixed pair (*l, k*), and the plotted curves show the radial dependence of $\tilde{\Delta}(x)$ for various values of α_0.

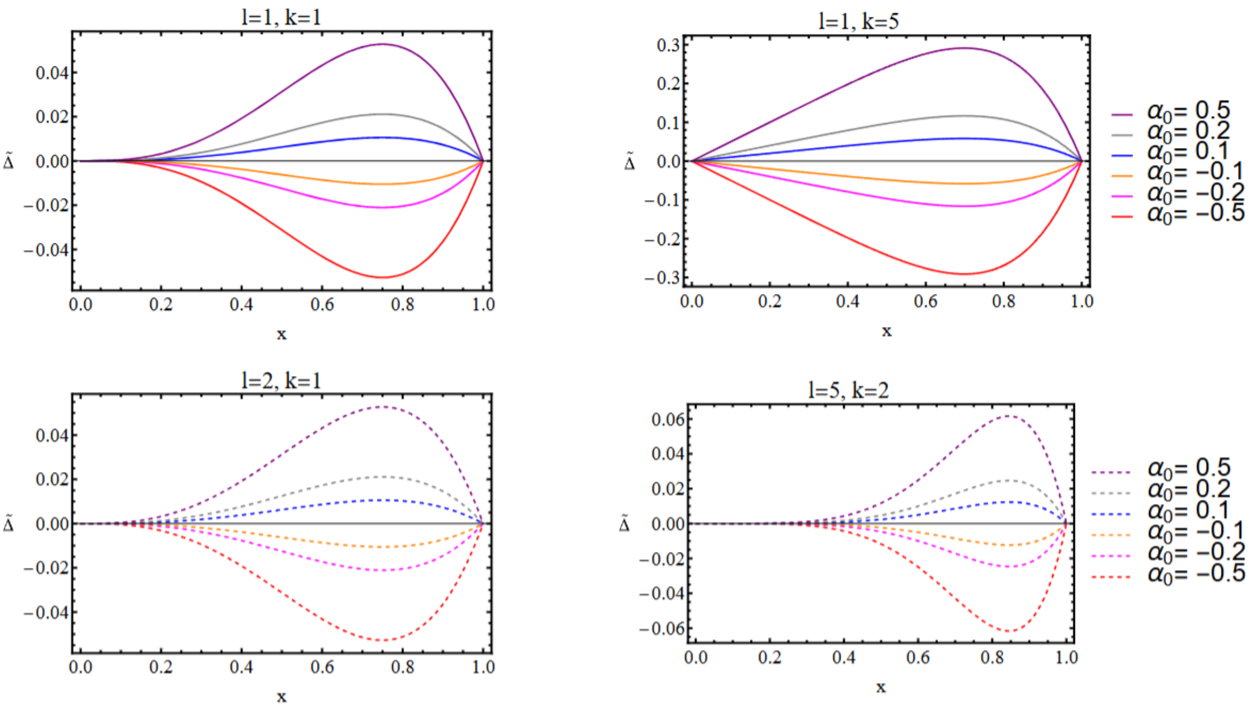

Figure 1 – Radial variation of the anisotropy parameter $\tilde{\Delta}(x)$ for different values of α_0 and combinations of l and k. Each subplot corresponds to a fixed (*l, k*), with curves showing variations in α_0

As shown in Figure 1, when $l > 0$ and $k > 0$, the anisotropy profile is smooth and bounded across the domain. The function peaks at an intermediate radius and naturally vanishes at the boundaries x = 0 and x = 1, consistent with physical expectations for equilibrium models.

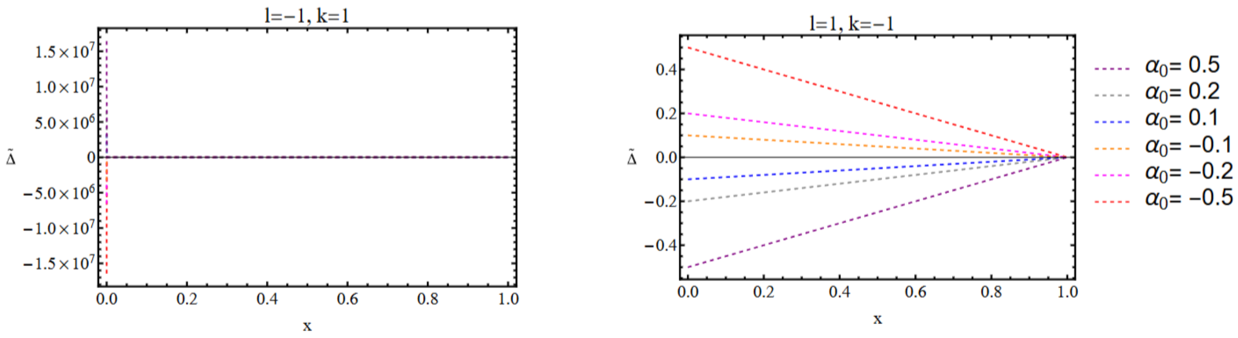

Figure 2 – Radial profiles of $\tilde{\Delta}(x)$ for fixed $l = k = -1$, illustrating nonphysical behavior. Curves with $l = -k$ are shown for varying values of α_0

In contrast, parameter combinations with negative values of l or k, such as $l = -1$ or $k = -1$, lead to pathological behavior, including steep gradients and unbounded profiles. These are visualized in Figure 2. While the boundary conditions are mathematically satisfied, the resulting profiles often include sharp internal spikes or asymmetries that may challenge physical viability.

Strong Anisotropy Regimes ($|\alpha_0| > 1$)

We extended the analysis to include stronger anisotropy values: $\alpha_0 = \pm 2, \pm 3, \pm 4$, to explore how extreme deviations from isotropy affect the radial structure. These values may be relevant in contexts involving strong magnetic fields or rapid rotation in compact stars. Figure 3 demonstrates that for $l > 0$, $k > 0$, the anisotropy profiles remain regular and bounded even for large magnitudes of α_0. The peak amplitude increases with $|\alpha_0|$, but the overall structure is preserved, making these configurations reliable for further physical modeling.

We also examined marginal cases such as $l = 3$, $k = -1$ and $l = 0.9$, $k = -0.5$, illustrated in Figure 4. These models satisfy the boundary conditions, yet their internal anisotropy gradients are significantly steeper, suggesting increased internal stresses. While mathematically consistent, these edge cases may correspond to unstable configurations and require further dynamical or perturbative analysis to confirm their viability.

Classification of Behavior

Now we can set the behavior of the anisotropic factor as follows:

• Regular cases: $l > 0$, $k > 0$ with moderate or strong α_0, producing smooth and bounded profiles.

• Edge cases: Mixed sign parameters or small positive l, k, resulting in steep but bounded internal gradients.

• Irregular cases: Negative l and/or k, yielding divergent or highly asymmetric profiles.

This classification serves as the basis for determining physically acceptable parameter space regions that can be used in equilibrium modeling and observation prediction for anisotropic compact stars.

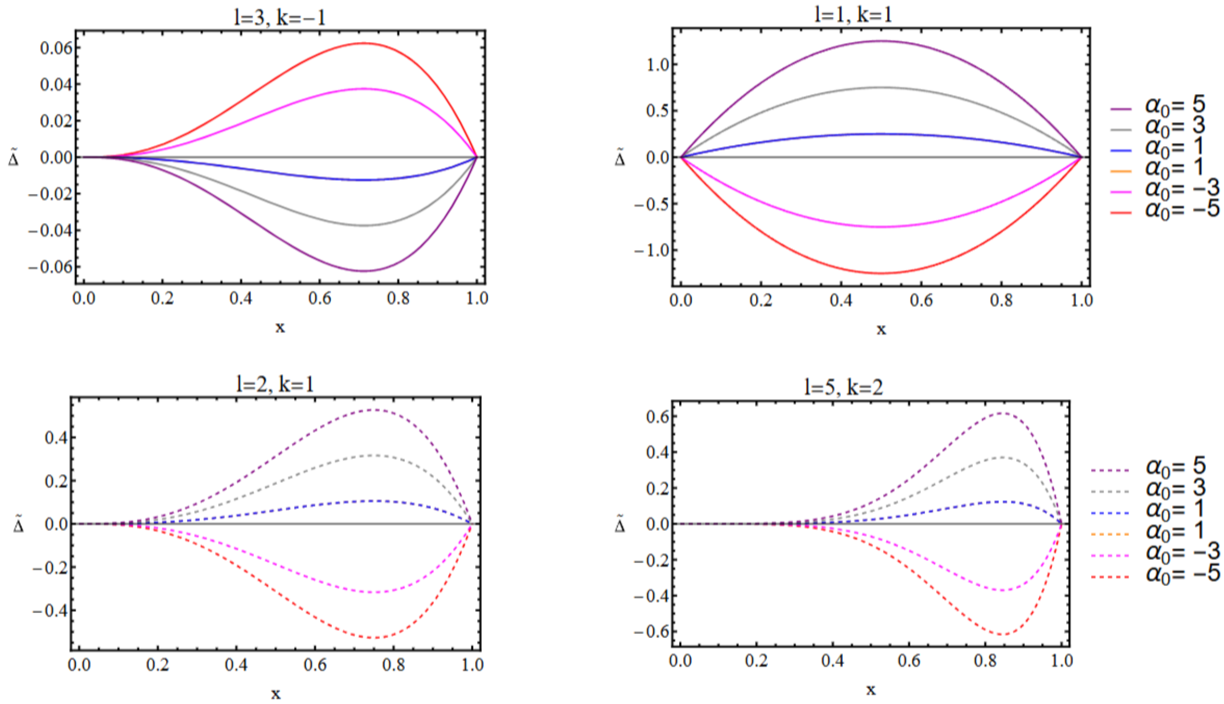

Figure 3 – The anisotropy profiles with positive $l > 0$ and $k > 0$ for large magnitudes of α_0

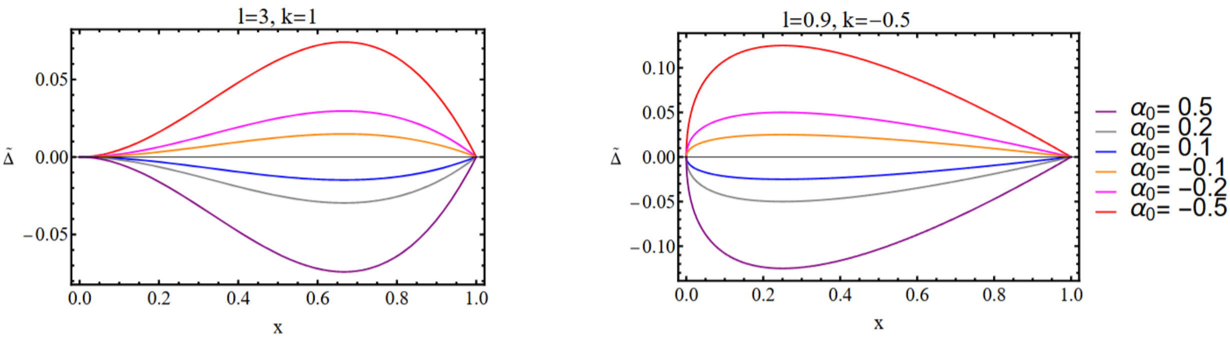

Figure 4 – The behavior of the dimensionless anisotropy function for strong regime α_0, l, k, for positive values of $l > k > 0$

The findings above suggest that pressure anisotropy influences how matter is distributed inside compact stars. Even simple deviations from isotropic conditions can shift pressure gradients, which can affect stellar stability and lead to changes in global characteristics such as mass and radius [19–22].

Discussion and Summary

This paper presented an analysis of how extended anisotropic pressure modifies the internal equilibrium of compact stars. We introduce a generalized anisotropic factor defined by three parameters: α_0 which sets the strength of anisotropy, and the shape parameters l and k which determine its radial profile. The model is constructed to satisfy standard boundary conditions and to allow flexible control over how anisotropy changes with radius.

The numerical study shows that when both shape parameters take positive values, the pressure inside the star forms a regular and physically reasonable pattern. In this regime, the pressure is nearly isotropic at the center and surface, while a clear maximum appears at an intermediate radius. Such a trend agrees with what might be expected from slow rotation or magnetic effects, suggesting that the model can represent realistic stellar configurations.

When the anisotropy strength is increased, the amplitude of pressure differences also grows. Yet, for positive parameter values, the overall pressure profile remains well behaved even in strongly anisotropic conditions. This outcome implies that white dwarfs are able to sustain stable internal structures provided that anisotropy varies smoothly with radius.

In contrast, if either l or k takes negative values, the pressure distribution becomes asymmetric, and sharp internal variations can arise. Although these solutions meet mathematical conditions, the resulting steep gradients may point to possible physical instabilities or regions of localized internal pressure.

These inputs are essential for solving the TOV equation and the mass continuity equation within a consistent and physically meaningful framework. Earlier investigations have shown that, to first order in the quadrupole moment, the gravitational field produced by a slowly rotating compact object can exhibit refractive characteristics analogous to those of a static, anisotropic deformed source[23–25]. This correspondence implies that, under specific physical conditions, pressure anisotropy within white dwarfs may replicate the effects typically attributed to slow rotation, particularly in terms of their influence on gravitational lensing and the deflection of light rays [26–28]. Exploring this analogy offers a valuable framework for identifying observational differences between anisotropy-induced and rotation-induced deformations [29, 30]. This allows for the modeling of a broad class of physically realistic configurations within a consistent framework and facilitates comparison with observational constraints [31–33].

The findings provide a coherent basis for examining how pressure anisotropy affects the internal balance and stability of compact stars. The approach developed in this work unifies the description of anisotropy within a single formulation that can be applied in future relativistic simulations.

In addition to the theoretical analysis, it is helpful to compare the obtained results with available observational data. The calculated range of the surface gravitational redshift, $z_s \sim 10^{-4} - 10^{-3}$, falls within the values reported for several massive and strongly magnetized white dwarfs observed in optical and ultraviolet bands. These measurements show similar trends: stronger internal stresses are associated with slightly lower redshift values. This agreement suggests that the model captures key features relevant for real compact stars, and the dependence of the redshift on the anisotropy parameter may serve as a simple observational indicator when interpreting spectroscopic data.

Moving forward, this generalized framework can be extended to different categories of compact objects, including neutron and quark stars, or other exotic configurations. In subsequent studies, it may be combined with realistic physical mechanisms that generate anisotropy – such as magnetic stresses or differential rotation – to analyze stability through both perturbative and time-evolution methods.

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