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Magnetic stabilization of the rotational motion of the nanosatellite in inclined orbit

Abstract. In this paper the problem of passive magnetic stabilization of the rotational motion of the nanosatellite in inclined orbit is considered.The effect of the gravitational torque on this stabilization is taken into account.
Passive magnetic stabilization allows the nanosatellite to stabilize by keeping one axis of the spacecraft aligned with the field lines of the Earth magnetic field in orbit. It is assumed that the geomagnetic field is modeled by direct dipole.
Rotational motion of nanosatellite is described by dynamic and kinematic Euler equations, which are solved by the fourth-order explicit Runge-Kutta method.
The results of computational experiments show that for orbits of inclinations over $i = 15^0$, passive magnetic stabilization is not effective to stabilize nanosatellites.
An analysis of obtained numerical results show that an influence of the geomagnetic field increases for the polar orbit. It is shown that for near equatorial orbits of inclinations under 15^0, passive magnetic stabilization is most effective to stabilize nanosatellites. The results of computational experiments show that for orbits of inclinations over 15^0, passive magnetic stabilization is not effective to stabilize nanosatellites and the perturbations caused by the gravitational torque tends to increase. In this way, this technique is not enough effective for satellite's orbits with an inclination of more 15^0. In order to achieve desired stabilization, one needs to take into account damping moments.
Key words: Passive magnetic stabilization, nanosatellite, rotational motion, geomagnetic field, direct dipole model, inclined orbit.

Introduction

During the last decades the small satellites or nanosatellites have become important tools for space research and space exploration with their strong merits of low-cost and quick development and have played an important role in the technological development. These features make nanosatellites very attractive and cost-effective materials for space education and research.

It is well known that the magnetic moment of interaction between the nanosatellite and the geomagnetic field may be used for the attitude stabilizationof a nanosatellite. Such magnetic control systems may be successfully used on the satellitesworking for a long time and don'tneed for the consumption of some working material.

Among existing stabilization systems, a passive magnetic stabilization systems have a special place and have become widely used to stabilize nanosatellites, since they are an exceptional reliable, energy efficient, relatively lightweight and are easy to manufacture.

Passive magnetic stabilization allows the nanosatellite to stabilize by keeping one axis of the spacecraft aligned with the field lines of the Earth magnetic field in orbit. It is expected that perturbation torques tending to produce turning moments about the center of mass of an orbiting nanosatellite will arise from many different causes. Among the more important isEarth's gravitational torque. As it was shown in [1] the gravitational torque is the major disturbance torque and exceeds the other disturbance. For this reason, in the considered mathematical model the passive magnetic stabilization technique is used by taking into account the effect of the gravitational torque. This model has been announced in [2].

Active three-axis magnetic attitude stabilization of a low Earth orbit satellite is considered in [3]. Analytical models and some exact solutions of the problem of the magnetic stabilization related to mathematical models of rotational motion of the satellite has been derived in [4]. In [5] an analytical model for the passive magnetically controlled attitude dynamics of the RAX nanosatellite is analyzed. Some with numerical simulations are derived to assess the properties of the attitude dynamics. The control is created by interaction between the magnetic moment generated by magnetorquers mounted on the satellite body and the geomagnetic field. This problem is quite complex and difficult to solve. Stabilization as a control problem for a satellite is considered in [6], where the control is carried out by a magnetic moment of current coils mounted on the satellite body. Using nonlinear models of hysteresis behavior and nonlinear model of spacecraft attitude dynamics, the problems related to design and analysis of passive stabilization have been considered in [7]. Here some results of nonlinear and quaternions-based mathematical model for satellite motion involving permanent magnets and hysteresis effects are presented.

In this paper we discuss the problem of passive magnetic stabilization of the rotational motion of the nanosatellite. The effect of the gravitational torque on this stabilization is taken into account.The different inclined orbits are considered in the geomagnetic field which is simulated by the direct dipole model.

Mathematical formulation of the problem

The problem of passive magnetic stabilization of the rotational motion of the nanosatellite is studied with into account the effect of the gravitational torque. In the this mathematical model we will discuss the rotational motion on inclined orbit around the Earth's center, and shall base the discussion on Euler's dynamical equations. To determine the satellite attitude in space we use the following reference frames. $O_a\overline{X}\overline{Y}\overline{Z}$ is the geocentric reference frame, O_a is the Earth center, $O_a\overline{Y}$ axis is directed along the Earth spin axis, $O_a\overline{Z}$ lies in the Earth equatorial plane and is directed to the point of the Vernal Equinox, $O_a\overline{X}$ axis is directed so the system to be a right-handed. *Oxyz* is the orbital reference frame, *O* is the nanosatellite center, *Oy* lies in the orbital plane, is perpendicular to the radius vector and directed as the orbital velocity does, *Oz* axis is directed along the radius-vector of thenanosatellite, *Ox* is directed such that the reference frame is right-handed. $O\overline{x}\overline{y}\overline{z}$ is the moving reference system, its axes are directed along the principal axes of inertia of the nanosatellite. We will define the attitude of the moving reference system $O\overline{x}\overline{y}\overline{z}$ with respect to the orbital reference frame *Oxyz* via the angles Ψ, Θ and Φ.

The mutual orientations of these reference frames are determined by the following linear transformations with direction cosine matrices:

$$\begin{array}{cccc} & x' & y' & z' \\ x & \alpha_1 & \alpha_2 & \alpha_3 \\ y & \beta_1 & \beta_2 & \beta_3 \\ z & \gamma_1 & \gamma_2 & \gamma_3 \end{array} \qquad \begin{array}{cccc} & x & y & z \\ \overline{X} & a_1 & a_2 & a_3 \\ \overline{Y} & b_1 & b_2 & b_3 \\ \overline{Z} & c_1 & c_2 & c_3 \end{array} \tag{2.1}$$

Let $u = \omega_\pi + \nu$ be the argument latitude, ω_π be the argument of periapsis, ν be the true anomaly, i be the orbit inclination and Ω be longitude of the ascending node of the orbit from the point of the Vernal Equinox. We will use the following relationships between the direction cosines, the angles Ψ, Θ, Φ and the above defined parameters [8]:

$$
\begin{cases}
\alpha_1 = \cos\Theta\cos\Phi, \\
\alpha_2 = -\cos\Theta\sin\Phi, \\
\alpha_3 = \sin\Theta, \\
\beta_1 = \cos\Psi\sin\Phi + \sin\Psi\sin\Theta\cos\Phi, \\
\beta_2 = \cos\Psi\cos\Phi - \sin\Psi\sin\Theta\sin\Phi, \\
\beta_3 = -\sin\Psi\cos\Theta, \\
\gamma_1 = \sin\Psi\sin\Phi - \cos\Psi\sin\Theta\cos\Phi, \\
\gamma_2 = \sin\Psi\cos\Phi + \cos\Psi\sin\Theta\sin\Phi, \\
\gamma_3 = \cos\Psi\cos\Theta.
\end{cases}
\qquad
\begin{cases}
a_1 = -\sin u\sin\Omega + \cos u\cos\Omega\cos i, \\
a_2 = -\cos\Omega\sin i, \\
a_3 = \cos u\sin\Omega + \sin u\cos\Omega\cos i, \\
b_1 = \cos u\sin i, \\
b_2 = \cos i, \\
b_3 = \sin u\sin i, \\
c_1 = -\cos\Omega\sin u - \sin\Omega\cos u\cos i, \\
c_2 = \sin\Omega\sin i, \\
c_3 = \cos\Omega\cos u - \sin\Omega\sin u\cos i.
\end{cases}
\tag{2.2}
$$

The kinematic and dynamic equations of the angular motion of the nanosatellite in an inclined orbit is described by

$$
\begin{cases}
\dot{\Phi} = p - (p\cos\Phi - r\sin\Phi)tg\,\Phi - \dfrac{\sin\Psi}{\cos\Theta}\dfrac{\omega_0}{(1-e^2)^{3/2}}(1+e\cos\nu)^2, \\
\dot{\Psi} = \dfrac{(p\cos\Phi - r\sin\Phi)}{\cos\Theta} + \sin\Psi tg\Theta\dfrac{\omega_0}{(1-e^2)^{3/2}}(1+e\cos\nu)^2, \\
\dot{\Theta} = q\sin\Phi + r\cos\Phi + \cos\Psi\dfrac{\omega_0}{(1-e^2)^{3/2}}(1+e\cos\nu)^2,
\end{cases}
\tag{2.3}
$$

$$
\begin{cases}
J_{\bar{x}}\dot{p} + (J_{\bar{z}} - J_{\bar{y}})qr = M_{m\bar{x}} + M_{g\bar{x}}, \\
J_{\bar{y}}\dot{q} + (J_{\bar{x}} - J_{\bar{z}})pr = M_{m\bar{y}} + M_{g\bar{y}}, \\
J_{\bar{z}}\dot{q} + (J_{\bar{y}} - J_{\bar{x}})pq = M_{m\bar{z}} + M_{g\bar{z}},
\end{cases}
\tag{2.4}
$$

where

$$
\dot{\nu} = \frac{\omega_0}{(1-e^2)^{3/2}}(1+e\cos\nu)^2. \tag{2.5}
$$

Here $J_{\bar{x}}, J_{\bar{y}}$ and $J_{\bar{z}}$ are the components of principal moment of inertia, *p, q* and *r* are the projections of the nanosatellite's angular velocity onto the axis of the moving reference system $O\bar{x}\bar{y}\bar{z}$. Further, $M_m = (M_{m\bar{x}}, M_{m\bar{y}}, M_{m\bar{z}})^T$ and $M_g = (M_{g\bar{x}}, M_{g\bar{y}}, M_{g\bar{z}})^T$ are the magnetic and gravitational moments, respectively, ω_0 is the angular velocity of the orbital motion and *e* is the eccentricity.

The magnetic and gravitational moments $M_m = (M_{m\bar{x}}, M_{m\bar{y}}, M_{m\bar{z}})^T$, $M_g = (M_{g\bar{x}}, M_{g\bar{y}}, M_{g\bar{z}})^T$ will be calculated analytically [10].

Mathematical model of the magnetic field

According to the mathematical theory of magnetic field of Earth, the potential of the internal geomagnetic field U_m which varies inversely with the distance from Earth's center to a point in space, is represented by the following series spherical harmonics [9]:

$$
U_m = \sum_{n=1}^{\infty}\frac{R_e^{n+2}}{R^{n+1}}\sum_{m=0}^{\infty}P_n^m(\cos\theta_R)\left(g_n^m\cos m\lambda_R + h_n^m\sin m\lambda_R\right). \tag{3.1}
$$

Here R_e is the mean equatorial radius of the Earth, R, λ_R, θ_R are the spherical coordinates of the satellite in the space where the potential is calculated (R is the radius-vector of the nanosatellite, λ_R is the geographical longitude and θ_R is the angle between the radius-vector and the spin axis of the Earth), $P_n^m(\cos\theta_R)$ is the associate Legendre function of the first kind of degree n and order m, g_n^m and h_n^m are the Gaussian coefficients. Then the vector of geomagnetic field strength with potential (3.1) is defined as follows

$$\vec{H} = -\nabla U_m .$$

It is known that the Earth's magnetic field on the surface of the globe, as well as near-Earth space in the first approximation close to the field of a uniformly magnetized sphere or field of a dipole placed in the center of the Earth.The individual members of the series (3.1) allow some physical interpretations. If we take into consideration the coefficient g_1^0 only (*n=1, m=0*), the field described by the series (3.1) is the field of a dipole situated at the center of the Earth and oriented in the direction North to South. Such model of the geomagnetic field is called direct dipole.

According to the dipole field the vector $\vec{H}$ can be represented as follows [8]:

$$\vec{H} = \frac{\mu_e \{\vec{k}_E - 3(\vec{k}_E \cdot \vec{e}_E)\vec{e}_E\}}{R^3},$$

where μ_e is the Earth's magnetic moment, $\vec{k}_E$ is the unit vector of the dipole axis, $\vec{e}_E$ is the unit vector in the direction of the radius-vector. By means of projections of this vector on the geocentric reference system $O_a\overline{X}\overline{Y}\overline{Z}$ this yields :

$$\begin{cases} H_{\overline{X}} = -\dfrac{3m_e}{R^3}\sin i\cos i\sin^2 u, \\ H_{\overline{Y}} = \dfrac{m_e}{R^3}(1-3\sin^2 i\sin^2 u), \\ H_{\overline{Z}} = -\dfrac{3m_e}{R^3}\sin i\sin u\cos u. \end{cases} \tag{3.2}$$

Let us consider the simplest practical method of realizing passive satellite stabilization. This involves providing orientation along the vector $\vec{H}$ of the local geomagnetic field strength. In this case, a restoring magnetic torque, arise from the interaction between a permanent magnet installed on nanosatellite'sboard and the local geomagnetic field. The magnetic moment of the magnet is chosen to be sufficiently strong to govern the motion of the satellite with respect to the vector $\vec{H}$.

Within the framework of the direct dipole, the magnetic torque is defined as follows:

$$M_m = \vec{I}_0 \times \vec{H} \tag{3.3}$$

Here the vector $\vec{I}_0 = (I_{0\bar{x}}, I_{0\bar{y}}, I_{0\bar{z}})^T$ is the magnetic moment of the nanosatellite. We assume that the vector $\vec{I}_0$ is located along the $O\bar{z}$ axis, i.e. $\vec{I}_0 = (0,0,I_0)^T$. Note that this moment arises due to the permanent magnet on board. Using (2.1) and (2.2) in (3.2), after simple calculations, we find the projections $H_{\bar{x}}, H_{\bar{y}}, H_{\bar{z}}$, of the geomagnetic field strength vector $\vec{H}$ [10]. Then for the projections $M_{\bar{x}}, M_{\bar{y}}, M_{\bar{z}}$ of the magnetic torque (3.3) of the Earth we have: $M_{\bar{x}} = -I_0 H_{\bar{y}}$, $M_{\bar{y}} = I_0 H_{\bar{x}}$, $M_{\bar{z}} = 0$. Using projections $H_{\bar{x}}, H_{\bar{y}}, H_{\bar{z}}$ [10] we obtain:

$$\begin{cases} M_{\bar{x}} = -\dfrac{I_0\mu_e}{R^3}\{(\cos\Psi\sin\Phi + \cos\Phi\sin\Theta\sin\Psi)[(3(\cos^2 u + \cos i\sin^2 u)\sin u\sin\Omega \\ -(1+3\cos i\cos u\sin^2 u + \cos^2 i\cos u\sin^2 u - 3\sin^2 i\sin^2 u)\cos\Omega] - (\cos\Phi\cos\Psi \\ +\sin\Phi\sin\Theta\sin\Psi)(3(1+\cos i)\cos u\sin i\sin u - 1 + 3\sin^2 i\sin^2 u)\sin u \\ +[3\cos^2 u\cos\Omega\sin u - 3\cos i\cos\Omega\sin^3 u - 6\cos^2(i/2)\cos i\cos u\sin^2 u\sin\Omega \\ +(3\sin^2 i\sin^2 u - 1)\sin\Omega]\cos\Theta\sin\Psi\}\sin i, \\ M_{\bar{y}} = \dfrac{I_0\mu_e}{R^3}\{(3\cos i\cos\Omega\sin^3 u - 3\cos^2 u\cos\Omega\sin u + \sin\Omega - 3\sin^2 i\sin^2 u\sin\Omega \\ +6\cos^2(i/2)\cos i\cos u\sin^2 u\sin\Omega)\sin\Theta + [(3(1+\cos i)\cos u\sin i\sin u - 1 \\ +3\sin^2 i\sin^2 u)\sin\Phi\sin u - [(1+3\cos i\cos u\sin^2 u + 3\cos^2 i\cos u\sin^2 u \\ -3\sin^2 i\sin^2 u)\cos\Omega - 3(\cos^2 u + \cos i\sin^2 u)\sin u\sin\Omega]\cos\Phi]\cos\Theta\}\sin i, \\ M_{\bar{z}} = 0. \end{cases} \tag{3.4}$$

The gravitational torque $M_g = (M_{g\bar{x}}, M_{g\bar{y}}, M_{g\bar{z}})^T$ produced by the gravitational field of the Earth, has the form [2]:

$$\begin{aligned} M_{g\bar{x}} &= \frac{3\mu}{R^3}(J_{\bar{z}} - J_{\bar{y}})\gamma_2\gamma_3, \\ M_{g\bar{y}} &= \frac{3\mu}{R^3}(J_{\bar{x}} - J_{\bar{z}})\gamma_1\gamma_3, \\ M_{g\bar{z}} &= \frac{3\mu}{R^3}(J_{\bar{y}} - J_{\bar{x}})\gamma_1\gamma_2. \end{aligned} \tag{3.5}$$

Here $\mu = GM_E$ is the gravitational parameter of the Earth G is the gravitational constant and M_E is the mass of the Earth).

Numerical simulation of rotational motion of the nanosatellite in different inclined orbits

This section deals with the numerical solution of the coupled system of differential equations (2.3)-(2.5). We will use here the fourth-order explicit Runge-Kutta method, which is popular as each stage can be calculated with one function evaluation [11].

The results of computational experiments are shown in Fig. 1-2. Specifically, Figures 1 illustrate the behaviour of the projections p, q, r of the nanosatellite's angular velocity onto the axis of the moving reference system $O\overline{xyz}$. The first figure, corresponding to the inclination $i = 0$, show that stabilization of the angular velocity occurs when $t > 20000$ *sec*. The reason is that for equatorial orbit the strength vector $\vec{H}$ remains constant at all points of orbit and is orthogonal to the orbital plane. For other orbits of inclinations over $i = 15^0$, passive magnetic stabilization is not effective to stabilize nanosatellites, as next figures show. Similar effects for orientation angles Φ, Ψ, Θ are observed from Figure 2.

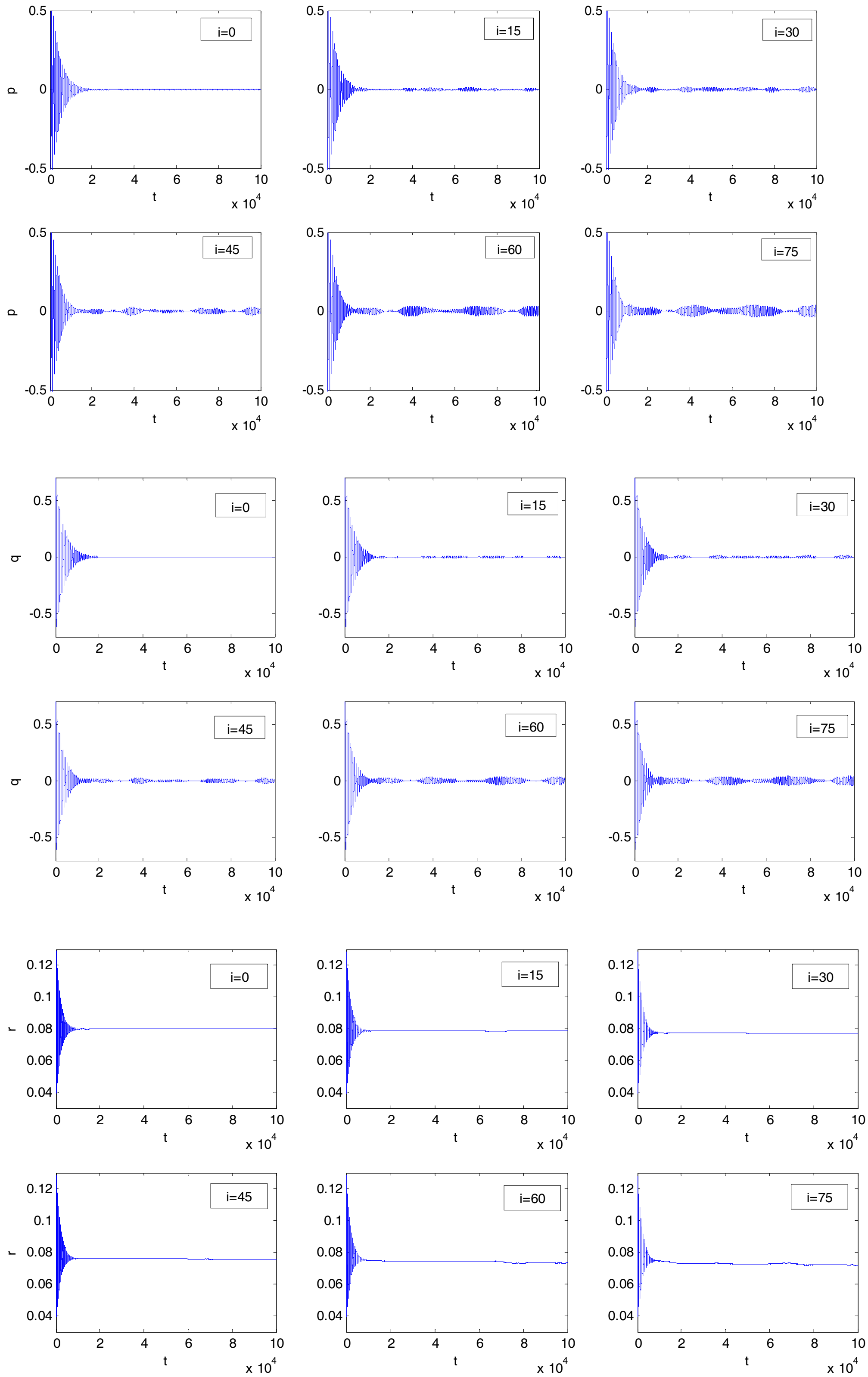

Figure 1 - Behavior of the projections *p, q, r* of the nanosatellite angular velocity for orbit inclinations $i = 0;\ 15^0;\ 30^0;\ 45^0;\ 60^0;\ 75^0$

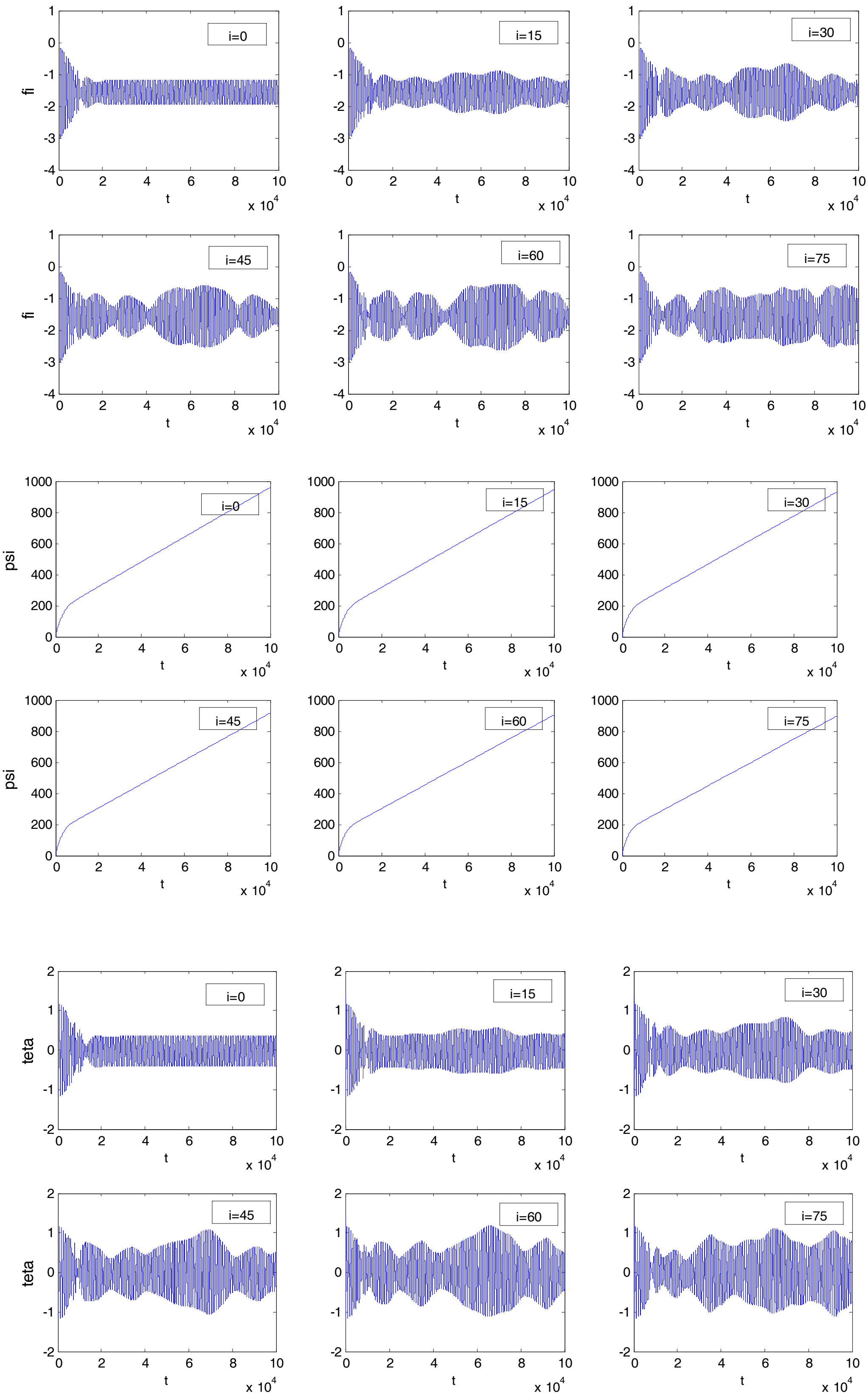

Figure 2 -Behavior of the orientation angles Φ, Ψ, Θ of the nanosatellite for orbit inclinations $i = 0$; 15^0; 30^0; 45^0; 60^0; 75^0

Conclusions

During the rotational motion of the satellite along its orbit, the magnitude and direction of the vector of the geomagnetic field strength is varying. This makes impossible to make a satellite's orientation as desired.

In this paper the stabilization of the nanosatellite on the inclined orbits in the geomagnetic field is considered. The geomagnetic field is modelled by direct dipole. The rotational motion of nanosatellite defined by the systems of differential equations (2.3)-(2.5). The problem of passive magnetic stabilization of the rotational motion of the nanosatellite is studied by taking into account the effect of the gravitational torque on this stabilization. An analysis of obtained numerical results show that an influence of the geomagnetic field increases for the polar orbit, since the strength vector varies at each point of the satellite's orbit. It is shown that for near equatorial orbits of inclinations under , passive magnetic stabilization is most effective to stabilize nanosatellites. However, this technique is not enough effective for nanosatellite's orbits with an inclination of more . In order to achieve desired stabilization, one needs to take into account damping moments.

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