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Nonlinear inverse problem of dinamics mix of gas based on the model of cloud formation

Abstract. The solution of the inverse problem is to restore the nonlinearity parameters based on some additional information. A new method is presented as an example of a nonlinear inverse problem for a model of the dynamics of the gas mixture based on the models of cloud formation. The unknowns are the parameters of the nonlinearity in the right-hand sides of equations of hydrodynamics. The solving of the inverse problem is to restore the nonlinearity parameters based on some additional information. Our method is presented as an example of a nonlinear inverse problem for a model of the dynamics of the gas mixture based on the models of cloud formation. The unknowns are the parameters of the nonlinearity in the right-hand sides of equations of hydrodynamics. On the basis of numerical experiments of solving of the direct problem identified flow characteristics that are sensitive to changes in the searching parameters. Then the data bank is formed on the grid of these parameters, which allows to find the best approximation of given set of characteristics.

Key words: nonlinear inverse problems, nonlinearity parameters, cloud water, reconstruction, microphysical processes, numerical solution, equations of hydrodynamics.

Introduction

Inverse problems in hydro and gas dynamics form a relatively new area of mathematical physics, have been applied in mechanics, technology, and are used in areas such as aerodynamics, hydrodynamics, filtration theory, the theory of the explosion. These problems often arise when trying to describe the characteristics of the environment in which take place various physical and chemical processes, the results of observations of these processes in an accessible area for measurements. For example, the study of inverse problem of recovering the right part for the Stokes equations discussed in [1], which also provides an overview of related work in this area. However, these works are theoretical. Numerical examples of the solution of inverse problems of source reconstruction for hydrodynamic problems we are currently not known.

We have considered the mathematical formulation of the nonlinear inverse problem of hydrodynamics on the case study of the dynamics of convective clouds with the microphysical processes in the two-dimensional formulation. In this formulation the unknowns are the parameters of the nonlinearity in the right-hand sides of equations for the water content of the cloud. A new method for the approximate solution of nonlinear inverse problems for the restoration of a finite set of nonlinearity parameters consists the following stages:

- to identify flow characteristics on the basis of systematic calculations of the direct problem that can be observed in the experiment and which are sensitive to changes in these parameters;
- to form a data bank on the grid of these parameters for each of the identified characteristics;
- to search a set of parameters for a given set of characteristics is carried out on the basis of best approximation on the grid.

It was found that the sensitivity characteristic of a change in microphysical parameters is cloud water content in the center and at the periphery of the formation field of clouds by the basis of systematic numerical experiments.

Numerical solution of the direct problem for the mix gas dynamics based on the model of cloud formation.

Modern models of formation and development of convective clouds are developed on the basis of a system of nonlinear integral-differential equations, allowing adequately take into account the dynamics of clouds and microphysical processes in it [2]. Analytical solution of such a system is not possible, the numerical solution on a computer is also facing a number of challenges. Limited possibilities are forcing researchers to simplify the initial system of equations based on the particular application of tasks and requirements for the simulation results.

Numerical models of clouds (steam – water) are classified according to the degree of completeness and detail of accounting hydrodynamic, thermodynamic and microphysical processes. Attempts of classification can be found in [3, 4]. The models describing the dynamics of clouds is divided into one-, two- and three-dimensional [5]. The degree of detailed consideration of microphysical processes may be different too.

A few models that take into account both dynamic and microphysical processes in equal measure are known [6,7] .

The main computational difficulties of modeling microphysical processes and large-scale dynamics due to the fact that the microphysical and dynamical processes have substantially different spatial and temporal scales and direct modeling of microphysical processes imposes very high demands on the ability of calculation grid to permit that. The most complete description of microphysical processes possible with fairly complex integro-differential equations [7] that can be solved only numerically. Moreover, the need to introduce many variables to adequately describe the solid phase makes finding a solution to these equations is almost impossible without major simplifications even on modern computing. To overcome these difficulties it was proposed to describe the microphysical processes by using the so-called parametric approach, the meaning of which is to replace the detailed description of microphysical processes by means of kinetic equations for the distribution of particle (water content cloud droplets, rain drops, etc.). In the model we used parameterized form of microphysical processes. It is believed that all the moisture in the cloud is composed of water vapor, cloud droplets, raindrops and – at the appropriate temperatures below freezing – particles of crystalline precipitation. Eligibility is based on such assumptions, experimental data on the microstructure of clouds and precipitation .

The developed model allows the calculation of the field humidity, clouds and precipitation based on parameterized microphysics of Kessler [8] with the known dynamic characteristics [9], taking into account of the three most important processes in this phenomenon, as the processes of coagulation, vaporization, and self-drops. Thus, the entire atmospheric moisture is presented in the form of three components: water vapor, cloud water and precipitation.

The initial system of equations for the temperature, specific humidity, specific conductivity of the cloud and precipitation water content will be as follows:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+w\frac{\partial T}{\partial z}=\frac{1}{\rho}\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)-L(\Delta+E_r)+\frac{dP}{dt} \tag{1}$$

$$\frac{\partial q_v}{\partial t}+u\frac{\partial q_v}{\partial x}+w\frac{\partial q_v}{\partial z}=\Delta+E_r+\Delta_b q_v \tag{2}$$

$$\frac{\partial q_c}{\partial t}+u\frac{\partial q_c}{\partial x}+w\frac{\partial q_c}{\partial z}=-\Delta-A_r-C_r+\Delta_b q_c \tag{3}$$

$$\frac{\partial q_r}{\partial t}+u\frac{\partial q_r}{\partial x}+w\frac{\partial q_r}{\partial z}=-E_r+A_r+C_r+\frac{1}{\rho}\frac{\partial \rho W_t q_r}{\partial z} \tag{4}$$

where

T – temperature, L – specific heat of condensation, q_{vs} (g / g) – saturation specific humidity, q_v (g / g) – specific humidity, q_c (g / g) – the specific conductivity of the cloud, q_r (g / g) – the specific conductivity of precipitation, E_r – takes into account evaporation of precipitation, A_r – treatment of cloud elements in sediments, C_r – capture cloud elements falling hydrometeors – (water droplets), W_r – gravitational speed of

falling rain, Δ - the rate of condensation of water vapor, $\Delta = \begin{cases} 0\,,\, npu \;\; q \le q_{vs}\,, \\ \dfrac{dq_{vs}}{dt}\,,\, npu \;\; q > q_{vs}\,. \end{cases}$

Row of the authors [10,11,12] introduced the concept of specific moisture content, which is defined as the sum of the specific humidity and the specific conductivity of the cloud, and solve the following system of equations

:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial z} = \frac{1}{\rho}\lambda\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) - L\left(\Delta + E_r\right) + \frac{dP}{dt} \qquad (5)$$

$$\frac{\partial q}{\partial t} + u\frac{\partial q}{\partial x} + w\frac{\partial q}{\partial z} = E_r - A_r - C_r + \Delta_b q, \qquad (6)$$

$$\frac{\partial q_r}{\partial t} + u\frac{\partial q_r}{\partial x} + w\frac{\partial q_r}{\partial z} = -E_r + A_r + C_r + \frac{1}{\rho}\frac{\partial \rho W_t q_r}{\partial z}. \qquad (7)$$

The advantage of this approach is a reduction in the number of initial equations, which are considered.

The formulation of initial and boundary conditions. At the initial time we have:

$$T = 1 - mz\,,\; q_v = RHq_{vs}\,,\; q_c = 0\,,\; q_r = 0, \quad 0 \le z \le 1\,, \qquad (8)$$

where RH – the given relative humidity, m – vertical temperature gradient. the values of the unknown quantities are determined in accordance with the initial conditions at the entrance to and the mild conditions were accepted for them at the exit. We have following conditions on the bottom wall:

$$T = 1\,, \quad q_v = RHq_{vs}\,, \qquad q_c = 0\,, \quad \frac{\partial\left(W_t + W\right) q_r}{\partial z} = 0. \qquad (9)$$

The system of equations (5) – (9) was solved numerically. The calculations were performed with the following range of parameters: E_0 = 0,16, 0,1 ≤ L_c ≤ 4,5; 0 ≤ RH ≤0,9. Computational grid consists of (201h61) cells with the steps h_x = 0,05, h_z = 0,05, and time step τ = *0,0025.* The series of numerical experiments carried out for the solution of the direct problem depending on the nonlinearity parameters. The influence of microphysical parameters as the self-cloud elements in precipitation, cloud capture elements falling hydrometeors, precipitation evaporation process of cloud formation. In Figure 1, shows the contours of cloud water content at different (coefficient of capture of cloud hydrometeors elements). The impact of microphysical parameters like cloud capture elements with falling hydrometeors, evaporation on process of cloud formation was analysed. The contours of cloud water content for different coefficient of capture of cloud hydrometeors elements were showed in figure 1.

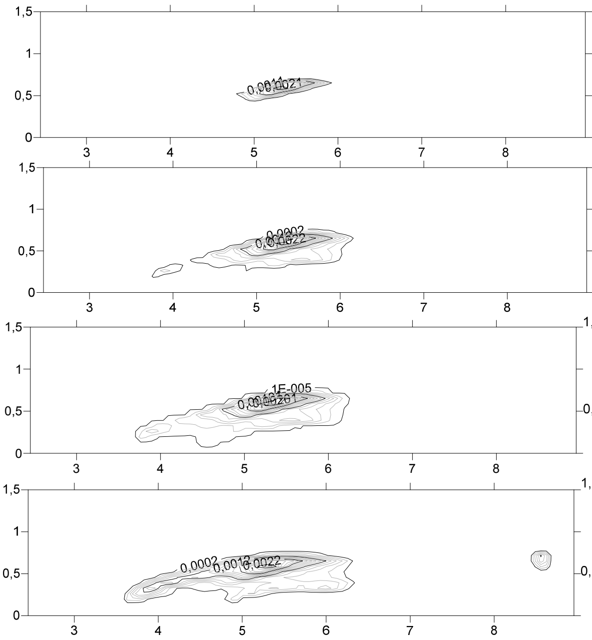

Figure 1-The value of cloud water content q_c for β =0.003 (a), β =3 (b), β =5 (c), β =7(d).

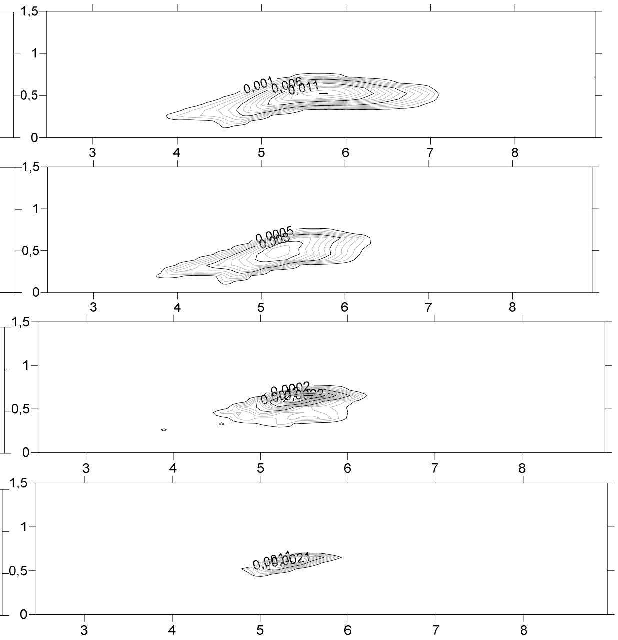

Figure 2-The value of cloud water content q_c for E =0.1 (a), E =3 (b), E =5 (c), E =7(d).

The contour lines of cloud water content for different coefficient of evaporation were showed in figure 2. As can be seen when β increases, the total value of cloud water content increases too, with growth of E it decreases, which corresponds to the physics of the process.

Formulation of the mathematical model in the form of inverse problems for dynamics of mix gas based on the model of cloud formation.

Analysis of the results of systematic calculations of the direct problem of cloud shows that there are observable characteristics that are monotonically related to the microphysical parameters of the flow. This allows us to formulate the inverse problem of reconstructing the microphysical parameters in the form:

Find the observable parameters of cloud formation, depending on the microphysics parameters of cloud formation and restore microphysical characteristics of the flow E and β by their values.

One important microphysical process of the atmosphere influencing the development of clouds and precipitation is coagulation, in particular gravitational coagulation, capture cloud elements falling hydrometeors. It is determined by the collision factor E that characterizes the capture efficiency at the coagulation growth of water droplets. It is known from experiments that for small drops which change in radius, capture efficiency is variable. According to the results of numerical calculations the mass of cloud water, which is convenient for observation value is sensitive to these parameters within certain ranges of selected coefficients. The results of numerical experiments at the clouds water and rainwater are shown in Table for availability for $0{,}1 \leq \beta < 50$ in the mean values of these parameters. The value of the coefficient of evaporation E was recorded in these calculations. The value of cloud water content in the central point cloud (the region of maximum values considered parameters) in dimensionless coordinates (8.75; 0.5) depended on empirical constants of the capture of cloud elements falling hydrometeors (water droplets) is monotone, which makes it possible to recover it.

Table 1 – Value clouds water and rain water at different β

(x,y)	β	0.1	0.5	1	3	5	7	10	50	
(3.9; .225)		.001483	.001484	.001484	.001484	.001484	.001484	.001638	.001490	q_c
		.000303	.000303	.000303	.000304	.000304	.000304	.000028	.000310	q_r
(3.9;0.3)		.000531	.000379	.000532	.000532	.000533	.000534	.000380	.000550	q_c
		.000031	.000000	.000031	.000031	.000031	.000031	.000000	.000030	q_r
(4.4; .425)		.000934	.001063	.000935	.000935	.000935	.000935	.001063	.000942	q_c
		.000292	.000027	.000292	.000293	.000293	.000293	.000027	.000295	q_r
(4.45; .35)		.002157	.003468	.002156	.002154	.002290	.002149	.003463	.002099	q_c
		.001175	.000164	.001175	.001175	.001892	.001175	.000165	.001175	q_r
(5.7; .425)		.006331	.011161	.006310	.006217	.006217	.006171	.011071	.005288	q_c
		.006120	.000919	.006123	.006133	.006133	.006139	.000952	.006238	q_r
(5.75;0.55)		.008213	.008196	.008175	.008092	.008010	.007929	.015801	.006425	q_c
		.009396	.009397	.009399	.009404	.009409	.009414	.001666	.009488	q_r
(6.4; . 3)		.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	q_c
		.003229	000596	.003228	.003225	.003222	.003219	.000616	.003152	q_r
(6.4; . 425)		.001507	.007054	.001482	.000000	.001376	.001324	.006855	.000495	q_c
		.010222	.001875	.010219	.010042	.010203	.010194	.001970	.009987	q_r

However, to verify that method works we should check it out on the so-called synthetic data. At first we conducted calculation of the direct problem for the given non-linearity parameters , then store the value of water content in the characteristic points as measured values. Let the value of the water content of the central part of the cloud is q_c^*=0.007970. Suppose, that q_c^* is the measured value, then, according to the data bank of these parameters β on the grid, we can obtain the value of the recovered parameters using simple interpolation:

$$\beta_{re\,cov\,er} = \beta_1 + \frac{(\beta_2 - \beta_1) * (q_c^* - q_1)}{q_2 - q_1},$$

где $q_c^* \in [q_1, q_2]$.

The ranges of microphysical parameters were chosen on the basis of numerical experiments and existing literature. They correspond to the area of the formation of clouds. According to the method proposed above, the coefficients of microphysical parameters such as β can be restored after the series of numerical experiments in the fields of its monotonic behavior.

Conclusions

Thus, we have performed a series of numerical calculations of the direct problem and analyzed the flow characteristics such as a full liquid water clouds and precipitation. It turned out that these parameters are not very sensitive to changes in the microphysical parameters and are not monotonic. Therefore, we used the value of cloud water content and water content of precipitation in the middle of the field of formulation of clouds as the observed cloud characteristics on which restore the required nonlinearity parameters. It was found that all these points clouds conductivity values depend monotonically on E and β. We have an example of restoration of β for a given E. Thus, the non-linear inverse problem with a finite set of unknown nonlinearity parameters can be formulated as the problem of solving a system of nonlinear equations of the form:

$$I_1\ (p_1, p_2, \ldots, p_n) = I_{10},$$

$$I_m\ (p_1, p_2, \ldots, p_n) = I_{m0},$$

m≥n, the measured (observed) values are in the right side of equations and nonlinearity parameters can be restored by them. This is the first study for the statement of the inverse problem with way showed above. The developed algorithms can be used for interpretation the measured data in technological and natural processes for simulation of the inverse problems for transport and diffusion processes with nonlinear sources, depending on the final set of parameters.

References

1. Oleg Yu Imanuvilov, Masahiro Yamamoto. Inverse source problem for the stokes system. Direct and Inverse Problems of Mathematical Physics. – P. Gilbert et al.(eds.). – 2000. – P. 441-451.
2. Mazin I.P., Shmeter S.M. Oblaka, stroenie i phisika obrazovania. – L.: Gidrometeoizdat, 1983. – P.280.
3. Kogan E.L. Sergeev B.N. Chvorostianov V.I. Chislennoe modelirovanie oblakov. // Trudy ZAO.- 1982. – №150.
4. Pastushkov R.C. Phiziko mathematicheskie modeli convectivnych oblakov // Trudy ZAO, 1982, 1973. – №112. – P. 3-14.
5. Silverman B.A., Glass M. A numerical simulation of warm cumulus cloud/ Part 1. Parameterized microphysics // J. Atm.Sci., 1973. – V.30. – №8. – P.1620-1637.
6. Takahashi T. Numerical of tropical showers // J. Atm.Sci., 1974. – V.31. – №1. – P.218-232.
7. Pruppacher H. R., Klett J. D. Microphysics of clouds and precipitation. – D. Reidel Publish. Co., 1978. – P. 714.
8. Kessler E. On the distribution and continuity of water substance in at mospheric circulations. – Amer. Meteor.Soc. Meteor. Monograph, 1969. – Vol. 10. – N 32. – P. 84.
9. Alieva B.K. Naimanova A.Zh. Smagulov Sh. S. Chislennoe issledovanie rotornych oblakov. // Vychislitelnye technologii. – Novosibirsk, 2003. – №1. – P.24-36.
10. Liu J.Y., Orville H.D. Numerical Modeling of precipitation and Cloud Shadow Effect on Mountain Induced Cumuli. // Journal of the Atmospheric Sciences. – 1969. – V.26. – №6. – P.1283-1298.
11. Orville H.D. A numerical study of the initiation of cumulus clouds over mountainous terrain // Journal of the Atmospheric Sciences, 1965. – V.22. – №6. – P.684-699.
12. Matveev L.T. Dynamica oblakov L., Gidrometeoizdat, 1981. – P.311.