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Simulation of resonant reactions in TTIK approach by C++ code

Abstract. We studied the interaction of ^{15}N with hydrogen at low energies using the Thick Target Inverse Kinematics (TTIK) method combined with the Time of Flight (ToF) technique. This combination enables a clear identification of different light recoil particles without compromising energy resolution in the extended gas target. The use of a gas target ensures minimal background and allows precise measurement of energy loss and reaction kinematics across a broad excitation function. At the same time, the ToF technique provides particle separation without degrading energy resolution. In this paper, we presented the results of theoretical calculations of energy losses in the gas target, implemented using a custom C++ code. The obtained results show reasonable agreement with experimental data. The simulations agree well with the experimental results for α -particles and ^{12}C nuclei, confirming the accuracy of the theoretical predictions for these reaction products. However, further improvements or theoretical refinements are required due to the insufficient fit observed in the proton-related calculations.

Keywords: TTIK, inverse kinematics, time of flight, resonant reaction, C++ simulation.

Introduction

The reactions $^{15}\text{N}(p,\gamma)^{16}\text{O}$ and $^{15}\text{N}(p,\alpha)^{12}\text{C}$ form a critical branch point in the CNO cycle [1–3]. These reactions have direct implications for stellar energy production and the nucleosynthesis of carbon, nitrogen, and oxygen isotopes [4–7]. Accurate modeling of these processes requires precise knowledge of the corresponding reaction rates, which in turn depend on a detailed understanding of the resonance structures dominating the cross sections at low energies. Direct measurements of these cross sections at stellar energies are impractical due to the rapidly decreasing Coulomb penetrability, necessitating extrapolations from higher-energy data supported by validated reaction models.

Previous studies of the $^{15}\text{N}(p,p)^{15}\text{N}$ elastic scattering reaction, notably those by Hagedorn [8] and Bashkin *et al.* [9], provided valuable early measurements of the excitation function over limited energy and angular ranges. However, these datasets are constrained by sparse angular coverage, lack of tabulated numerical data, and absence of statistical

uncertainties, complicating R-matrix analyses and reducing the reliability of extrapolated reaction rates.

Subsequent work at the University of Notre Dame's Nuclear Science Laboratory [10] extended measurements of the $^{15}\text{N}(p,p)^{15}\text{N}$ excitation function from 0.6 to 1.8 MeV using a 4-MV KN Van de Graaff accelerator. In this experiment, a well collimated proton beam was incident on a ^{15}N gas target, and scattered particles were detected by an array of five silicon detectors positioned between 90° and 165° in the laboratory frame. Although this setup enabled the study of both $^{15}\text{N}(p,p)^{15}\text{N}$ and $^{15}\text{N}(p,\alpha_0)^{12}\text{C}$ reactions, it did not allow for clear separation of α_1 particles from the $^{15}\text{N}(p,\alpha_1)^{12}\text{C}$ channel.

The thick-target inverse kinematics (TTIK) technique, combined with time-of-flight (ToF) measurements, has enabled improved identification of light recoils from different reactions as elastic, inelastic scattering, and nuclear reaction, even when using extended gas targets [11]. Accurate modeling of the energy loss of the heavy-ion beam through the gas target is crucial for data analysis in TTIK experiments [11].

Accurate simulations are therefore essential for the interpretation of TTIK experimental data. In this study, we developed a C++ program to simulate the time of flight and energy loss of particles in the TTIK method. The code models the propagation of particles through the gas target, accounting for continuous ionization losses and reaction kinematics. This allows for accurate reproduction of time distributions for various reaction channels and detector configurations. We applied this simulation framework to the $^{15}\text{N} + \text{p}$ interaction, including elastic and inelastic scattering, as well as the $^{15}\text{N}(\text{p},\alpha)^{12}\text{C}$ reaction, and compared the theoretical predictions with experimental observations.

We applied the Thick Target Inverse Kinematics (TTIK) approach to investigate the resonant reaction of $^{15}\text{N}(\text{p},\text{p})^{15}\text{N}$ and $^{15}\text{N}(\text{p},\alpha)^{12}\text{C}$ at the DC-60 cyclotron (Astana, Kazakhstan) [12]. Astana cyclotron accelerates ions from Li to Xe with energy 1.75 MeV/A, which was designed for material science studies and educate students. The parameters of DC60 cyclotron are limited to study resonant reaction by classical approach, whereas they are compatible with the TTIK approach [13]. The TTIK technique works in inverse kinematics [14-16]. The ion beam enters the scattering chamber through a thin entrance window typically titanium or havar foils. The chamber is filled with a high purity gas which works as a target and beam degrader. The beam loses energy in the extended gas target due to its energy losses. The gas pressure is adjusted such that the beam is fully stopped before reaching the detectors, which are mounted along the beam axis at the back of the chamber.

The light recoils as protons and α particles from the interaction are detected by a silicon detector array positioned at different angles including 0 degree in laboratory frame. Because the energy losses of these light particles in the gas are significantly lower compared to accelerated heavy ions, they successfully penetrate the gas volume and reach the detectors.

TTIK method has several advantages compared to the classical approach: single run, clear gas target and measurement at 180 degrees.

First, the entire excitation function of the reaction can be measured in a single run using a fixed beam energy, as the heavy ion slows down continuously while passing through the thick gas target. This allows the measurement of a wide energy range in one experiment, without the need to adjust the beam energy for each point. Second, the use of a pure gas

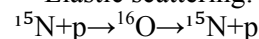
target provides a clean environment without the complications of target backing materials, reducing background signals. Finally, resonance scattering measurements at 180 degrees in the center-of-mass frame (which corresponds to 0 degrees in the lab frame) are especially important because Rutherford scattering is minimal there, making it easier to observe and study all possible resonances [13].

The ToF method was integrated into the TTIK setup to separate different particle types and various excitation states within the same reaction channels. By measuring the time between the cyclotron's RF signal and the signal from the detector, we can determine the velocity of the particle, which helps to identify its mass. This allows us to distinguish between different reaction channels that may have similar energies but produce particles with different speed keeping acceptable energy resolution.

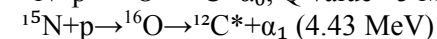
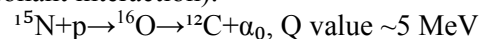
The calculated time is determined by the particle's passage through the gas target. It is measured from the moment the beam enters the gas until the recoil particle (e.g., proton, alpha particle, etc.) reaches the detector. Since the time depends on how long the particle travels through the gas, it is important to accurately calculate the ionization energy losses. Precise determination of these losses both for the beam and for the recoil particle is essential for correct time calculations and forms the basis of our theoretical modeling (see Chapter 2).

In our experiment, a ^{15}N beam with an initial energy of 26.85 MeV enters the scattering chamber through a 2.0 μm thin titanium window, it is losing about 3.95 MeV and entering the hydrogen gas target at 22.9 MeV (See fig. 1). The gas pressure is set at 218 Torr allowing stopped the beam before the detector. Reactions such as elastic and inelastic scattering, as well as $^{15}\text{N}(\text{p},\alpha)^{12}\text{C}$, can occur, each with distinct Q-values and kinematics.

- Elastic scattering:



- Inelastic scattering or nuclear reactions (e.g., resonant interaction):



These differences are used to distinguish processes in energy and time spectra. Fifteen silicon detectors ($10 \times 10 \text{ mm}^2$, 350 μm thick) are positioned at the back of the chamber, covering a wide angular range, including zero degrees. Light recoil particles, like protons and α -particles, lose less energy in the gas and are reliably detected.

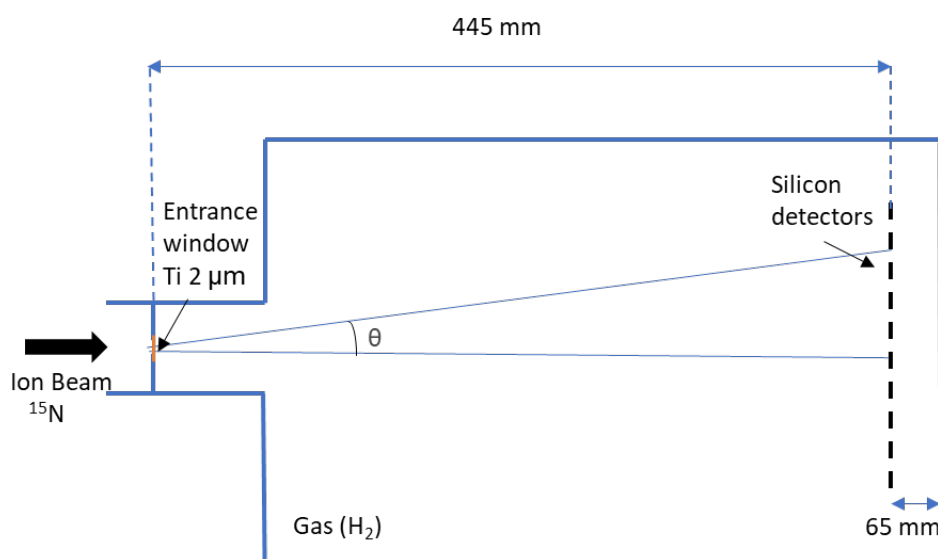


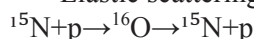
Figure 1 – Scheme of TTIK approach

In this study, we have written a C++ program to simulate the time and energy loss of particles in the TTIK method. The code models particle movement through the gas target, accounting for ionization losses and reaction kinematics. This allows for acceptable simulation of ToF distributions for various reaction channels and detector configurations. We applied this program to the $^{15}\text{N} + \text{p}$ interaction, including elastic and inelastic scattering, as well as the $^{15}\text{N}(\text{p}, \alpha)^{12}\text{C}$ reaction, to compare theoretical predictions with experimental data.

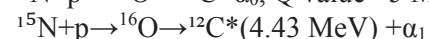
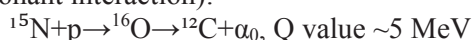
Method for calculating energy loss and time of flight in an extended gas target using C++ code

We have developed a C++ program to simulate the time and energy loss of particles within TTIK method. The code models the transport of a heavy ion beam, such as ^{15}N , through a gas-filled chamber, accounting for ionization losses and reaction kinematics. This allows for adequate simulation of time distributions for various reaction channels

- Elastic scattering:



- Inelastic scattering or nuclear reaction (e.g., resonant interaction):



By comparing these simulations with experimental data, we can better understand the underlying nuclear processes and improve particle identification in TTIK experiments.

The program performs a stepwise simulation of the beam's transport from the entrance window to the point where it stops in the gas. The reaction point is starting from the entrance of the beam into the gas medium and up to the final stopping point. At each small step Δx , the energy of the ^{15}N beam is updated using ionization loss data.

- The time of the beam from the entrance to the reaction point

- The time of the emitted product (e.g., proton or α -particle) from the reaction point to the detector

The simulation takes into account the geometry of the experimental scattering chamber, including

- The distance between the entrance window and the central detector,

- The positions and angles of silicon detectors,

These geometric factors are essential for correctly computing the flight paths and corresponding time values, especially when particles are emitted at non-zero angles. Thus, we obtain a complete energy and time dependence for various reaction depths and angles.

The C++ code was developed in a modular structure, providing flexibility for different reaction channels and detector geometries. The core of code has a loop that scans possible reaction points along the beam path, starting from the entrance window and progressing step by step through the gas target until the detector. For each step, the energy of the beam is updated based on ionization losses, and corresponding kinematic and time-of-flight

calculations are performed. There are several interlinked main modules included in this loop.

Ion Transport Module: In the first module, energy loss calculations for nuclear reactions in gas or solid targets are performed using stopping power data obtained from SRIM or LISE++ programs [17-20]. In our case, the gas option is used. The function estimates stopping power at a given energy by performing a linear interpolation algorithm. Then it calculates the energy loss by multiplying the interpolated stopping power with the small step size in distance (i.e., the thickness of the gas segment over which energy loss is computed). Four parameters are used in calculations: distance (from the entrance to the interaction point, and from the interaction point to the detector), energy of the particle according to loop (energies of beam and recoils), and two arrays from the SRIM/LISE++ files: the first contains tabulated energy values of the particle, and the second contains the corresponding stopping power values for those energies.

Kinematic Calculation Module: This module handles the kinematics of both elastic and inelastic scattering reactions. The function of the module requires as input the interacted nuclei masses, the masses of recoils, the lab angle of the detected particles, and the energy of the beam particle. Using these data, the module computes the laboratory energies of the outgoing particles for different processes like elastic (for example, $^{15}\text{N} + \text{p} \rightarrow ^{15}\text{N} + \text{p}$) and some inelastic reactions (for example, $^{15}\text{N} + \text{p} \rightarrow ^{12}\text{C} + \alpha_0/\alpha_1$) using non-relativistic two-body kinematic equations (3)–(16).

Time Calculation Module: This module estimates a particle's (beam and recoils) time of flight in the gas target. It requires four parameters: initial and final energy of the beam and recoils, mass of the particle, and the distance (from the entrance window to the reaction point and from the reaction point to the detector). The module calculates the average velocity over this segment based on energy loss and uses it to determine the travel time using formulas (17)–(20).

The Ion Transport Module calculates energy loss using the Bethe-Bloch formula, which explains how heavy charged particles lose energy as they ionize matter. The Kinematic and Time Calculation Modules apply conservation laws and non-relativistic kinematic equations. The next section will present the theoretical basis for these processes.

The position of the interaction point inside the gas depends on the stopping profile of the incoming

beam, which is governed by the Bethe-Bloch formula describing the ionization energy loss.

The slowing down of the incident ion beam before the reaction is described by the energy loss per unit length, given by the Bethe formula (for heavy charged particles in matter):

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} nZ \ln \frac{2m_e v^2}{I} \quad (1)$$

where, z — charge of the incident ion (e.g., 7 for ^{15}N), v — ion velocity, n — electron density of the target, Z — atomic number of the target material ($Z=1$ for hydrogen), I — mean excitation potential, e — elementary charge, m_e — electron mass. The input

data for ionization energy losses $\left(-\frac{dE}{dx}\right)$ of all particles in hydrogen gas are taken from external simulation tools such as SRIM or LISE++ . These data are provided in tabular form, typically listing energy values and corresponding stopping powers.

To obtain the stopping power at any intermediate energy value E , the program uses linear interpolation between two nearest data points (E_1, S_1) and (E_2, S_2), according to:

$$S(E) = S_1 + \frac{S_2 - S_1}{E_2 - E_1} (E - E_1) \quad (2)$$

where $S(E) = \left(\frac{dE}{dx}\right)_E$. This allows accurate tracking of energy losses even in fine steps.

To describe the kinematics of nuclear reactions, we use non-relativistic formulas for elastic collisions between an incident projectile and a target at rest in the laboratory frame. These expressions are derived from conservation of energy and momentum, and are valid when the projectile's velocity does not exceed 10% of the speed of light. Let M_1 and M_2 be the masses of the incident and target particles, respectively, and define the mass ratio as $x = \frac{M_1}{M_2}$

[21]. The incident particle has kinetic energy E_0 . After the collision, the scattered projectile and recoiling target have laboratory energies E_1 and E_2 , and are emitted at angles θ and ϕ , respectively.

The kinetic energy of the scattered projectile in laboratory frame is:

$$E_1 = E_0 \frac{\left(x \cos(\theta) + \sqrt{1 - x^2 \sin^2(\theta)} \right)^2}{(1+x)^2} \quad (3)$$

when $M_1 \leq M_2$.

$$E_1 = E_0 \frac{\left(x \cos(\theta) + \sqrt{1 - x^2 \sin^2(\theta)} \right)^2}{(1+x)^2} \quad (4)$$

when $M_1 > M_2$.

Laboratory energy of the recoil nucleus:

$$E_2 = E_0 \frac{4x \cos^2(\varphi)}{(1+x)^2} \quad (5)$$

For inelastic reactions, the incident particle M_1 and the target M_2 produce two reaction products: a light particle M_3 and a heavy particle M_4 . The reaction may release energy Q , defined as:

$$Q = (M_1 + M_2 - M_3 - M_4) \quad (6)$$

The total energy in the lab frame after the reaction is:

$$E_T = E_0 + Q = E_3 + E_4 \quad (7)$$

Angles and energies of the reaction products are determined using center-of-mass transformations and are governed by expressions involving coefficients A, B, C, D which relate the reaction kinematics to the masses and input energy. These coefficients satisfy:

$$A + B + C + D = 1 \quad (8)$$

$$A = \frac{M_1 M_4 (E_0 / E_T)}{(M_1 + M_2)(M_3 + M_4)} \quad (9)$$

$$B = \frac{M_1 M_3 (E_0 / E_T)}{(M_1 + M_2)(M_3 + M_4)} \quad (10)$$

$$C = \frac{M_2 M_3 (E_0 / E_T)}{(M_1 + M_2)(M_3 + M_4)} \left(1 + \frac{M_1 Q}{M_2 E_T} \right) \quad (11)$$

$$C = \frac{M_2 M_3 (E_0 / E_T)}{(M_1 + M_2)(M_3 + M_4)} \left(1 + \frac{M_1 Q}{M_2 E_T} \right) \quad (12)$$

When $B \leq D$ laboratory energy of the light product is given by:

$$E_3 = E_T B \left[\cos(\theta) + \sqrt{D / B - \sin^2(\theta)} \right]^2 \quad (13)$$

When $B > D$, E_3 is double valued,

$$E_3 = E_T B \left[\cos(\theta) \pm \sqrt{D / B - \sin^2(\theta)} \right]^2 \quad (14)$$

Laboratory energy of the heavy product when $A \leq C$:

$$E_4 = E_T A \left[\cos(\varphi) + \sqrt{C / A - \sin^2(\varphi)} \right]^2 \quad (15)$$

When $A > C$, E_4 is double valued,

$$E_4 = E_T A \left[\cos(\varphi) \pm \sqrt{C / A - \sin^2(\varphi)} \right]^2 \quad (16)$$

The total time measured in the experiment is the sum of two components:

1. **Time of the incident beam** from the entrance window to the reaction point, accounting for energy loss along the way

$$t_{beam} = \frac{d_1}{\bar{v}} \quad (17)$$

where, d_1 - distance from the entrance to reaction point, \bar{v} - average velocity, E_i, E_f - initial and final energy of beam, m_{nuc} - mass of nucleus.

$$\bar{v} = \frac{c \left[\sqrt{\frac{2E_i}{m_{nuc} c^2}} + \sqrt{\frac{2E_f}{m_{nuc} c^2}} \right]}{2} \quad (18)$$

2. **Time of the recoil product** (e.g., proton or ^4He) from the reaction point to the detector, also including energy loss due to ionization in gas:

$$t_{recoil} = \frac{d_2}{v} \quad (19)$$

where, d_2 – distance from the reaction point to detector.

Thus, the total time is given by:

$$t_{recoil} = t_{beam} + t_{recoil} \quad (20)$$

Results and discussions

Fig. 2 presents the two-dimensional histogram of deposited energy versus time for particles detected in the zero-degree laboratory frame. The x axis shows

energy in MeV and the y time in nanoseconds and the z axis (color scale) indicates the event counts. Three channels of the reaction are visible: protons (p) forming the lower band, upper band to alpha particles (α_0) associated with the ground state of ^{12}C and the middle band to alpha particles (α_1) corresponding to the excited state of ^{12}C at 4.43 MeV. Superimposed on the experimental data are theoretical curves for recoil protons, alpha particles all shown in red – calculated using non-relativistic two-body kinematics.

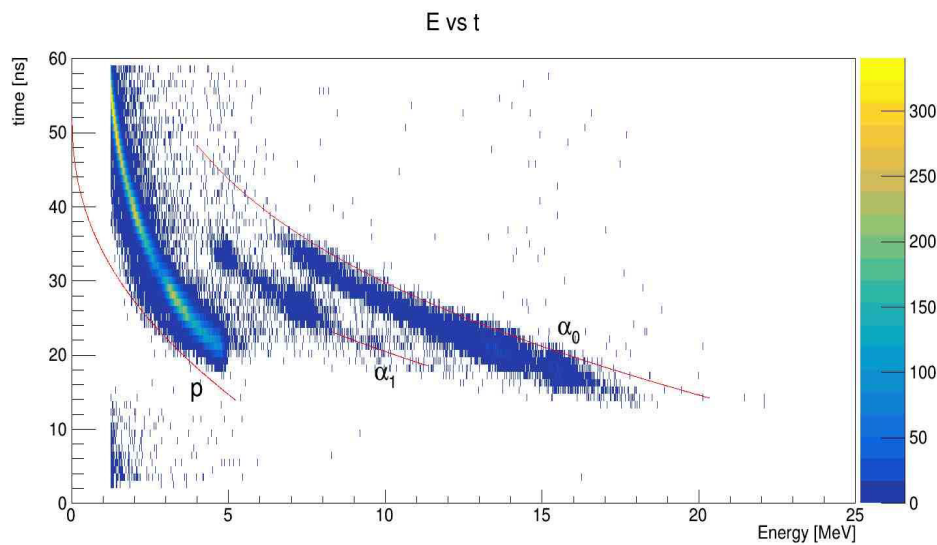


Figure 2 – Spectra for the products of the interaction of ^{15}N with hydrogen.

As can be seen from Fig. 2, the experimental and theoretical values generally demonstrate acceptable agreement. For α_0 , α_1 nuclei, the discrepancies do not exceed 15%. This means that the calculations of heavy particles accurately describe their energy loss and transport in the gas target. However, in the case of protons, they can reach up to 50% at individual energy points. This shows us the need for improved modeling of ionization energy losses for light particles.

The discrepancy observed between the experimental data and the theoretical curves time of flight for protons is primarily due to limitations in accurately modeling ionization losses for light particles in the gas target. Protons, being light particles, are sensitive to various factors, especially electron and nuclear interactions, which are difficult to accurately model. One of the solutions we plan to apply is to improve our computational models by

implementing more sophisticated approaches such as coupled channel methods and accurate stopping power calculations into the calculation code. One approach is to use modeling frameworks such as GEANT4 [22], which is promising because it allows for additional physical factors to be taken into account and provides more accurate modeling of energy loss and particle transport in complex materials. These improvements are aimed at providing more accurate modeling of energy loss processes. By applying these corrections, we can achieve better agreement between theoretical models and experimental data, leading to more reliable analysis of nuclear reactions involving light particles.

Conclusion

As a result of comparison between experimental data of energy and time (E vs t) with theoretical curves, the presence of protons and alpha particles as

products of the reaction $^{15}\text{N} + p$ was confirmed. For the α_1 and α_0 particles, there is good agreement between the theoretical curves and the experimental data. This indicates that the modeling and calculations for the energy losses are correct.

However, there are small deviations between the calculation and the experimental data for protons, due to the limitations of the modeling accuracy for light particles. We developed a transport code in C++ that simulates the time of flight through a gas target using ionization losses and kinematics within inverse kinematics method. The developed code allows us to simulate reaction processes in different target

materials and improve the precision of experimental measurements. Good agreement between calculations and experiment data supports validity of the basic model adopted for our simulation. Additional efforts will be made to improve the fit of the data at different energy values.

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References

1. Burbidge, E. M., Burbidge, G. R., Fowler, W. A., and Hoyle, F. "Synthesis of the Elements in Stars." *Reviews of Modern Physics* 29, no. 4 (1957): 547–650.
2. Bethe, H. A. "Energy Production in Stars." *Physical Review* 55, no. 5 (1939): 103–103.
3. von Weizsäcker, C. F. "Zur Theorie der Kernmassen." *Zeitschrift für Physik* 38 (1937): 176; 39 (1939): 633.
4. Gadioli, E., and Gadioli Erba, E. "Pre-Equilibrium Emission." *Nuclear Instruments and Methods* 146 (1977): 265–277.
5. Rolfstt, C., and Rodney, W. S. "Proton Capture by N at Stellar Energies." *Nuclear Physics* 235 (1974): 450–459.
6. Bray, R. H., Frawley, A. D., Opi, T. R., and Harbr, F. C. "Levels of ^{16}O Near 13 MeV Excitation from $^{15}\text{N} + p$ Reactions." *Nuclear Physics A* 288, no.2 (1977) :334–350.
7. O'Connell, W. J., and Hanna, S. S. "Giant E1 Resonance in ^{16}O Observed with the Reaction." *Physical Review C* 17, no. 3 (1978):892–902.
8. Hagedorn, F. B. "States in ^{16}O between 12.67 and 13.80 MeV." *Physical Review* 108, no. 3 (1957):735–742.
9. Bashkin, S., Carlson, R. R., and Douglas, R. A. "Cross Sections for Elastic Scattering and Reactions Due to Protons on ^{15}N ." *Physical Review* 114, no. 6 (1959): 1543.
10. deBoer, R. J., P. J. LeBlanc, S. Falahat, G. Imbriani, J. Görres, S. O'Brien, E. Uberseder, and M. Wiescher. "Elastic Scattering of Protons from ^{15}N ." *Physical Review C* 85, no. 3 (2012):1–7.
11. Nurmukhanbetova, A.K., Goldberg, V.Z., Nauruzbayev, D.K., ... Torilov, S.Y., Tribble, R.E. Implementation of TTIK method and time of flight for resonance reaction studies at heavy ion accelerator DC-60 . *Nuclear Instruments and Methods in Physics Research, Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*,847 (2017): 125–129
12. Gikal, B., Dmitriev, S., Apel, P., Bogomolov, S., Borisov, O., Buzmakov, V., Gulbekyan, G., Ivanenko, I., Ivanov, O., Itkis, M., Kazarinov, N., Kalagin, I., Kolesov, I., Papash, A., Paschenko, S., Tikhomirov, A., and Khabarov, M. "DC-60 heavy ion cyclotron complex: The first beams and project parameters", *Physics of Particles and Nuclear Letter*, 5 (2008): 642–644
13. Artemov, K. P., Belyanin, O. P., Vetoshkin, A. L., Wolski, R., Golovkov, M. S., Goldberg, V. Z., Madeja, M., Pankratov, V. V., Serikov, I. N., Timofeev, V. A., Shadrin, V. N., and Szmidler, J. "Effective Method of Study of α -cluster states", *Soviet Journal of Nuclear Physics* 52, 408 (1990):634–639
14. Hooker, J., G. V. Rogachev, V. Z. Goldberg, E. Koshchiy, B. T. Roeder, H. Jayatissa, C. Hunt, C. Magana, S. Upadhyayula, E. Uberseder, and A. Saastamoinen, "Structure of ^{10}N in $^9\text{C} + p$ Resonance Scattering" *Physics Letters B* 769 (2017): 62–66
15. Nurmukhanbetova, A.K., Goldberg, V.Z., Nauruzbayev, D.K., Volya A,Rogachev G.V, "R-matrix analysis of ^{22}Ne populated in $^{18}\text{O}(\alpha,\alpha)$ resonant elastic scattering", *Physical Review C* 109, no. 2 (2024): 1–5
16. La Cognata M., Fisichella M., Di Pietro A., Figuera P., Goldberg V. Z., Cherubini S., Fernández Garcia J. P., Gulino M., Lamia L., Lattuada D., Lattuada M., Pizzone R. G., Rapisarda G. G., Romano S., Sparta R., Spitaleri C., Torresi D., Tumino A., Zadro M.."Observation of $^{15}\text{N} + \alpha$ Resonant Structures in ^{19}F Using the Thick Target in Inverse Kinematics Scattering Method." *Physical Review C* ,99,034301 (2019):1–12
17. Hubert, F. "Stopping Power and Range Tables for Ions in Matter." *Atomic Data and Nuclear Data Tables* 46 (1990): 1–213.
18. Ziegler J. F., J. P. Biersack, and U. Littmark. *The Stopping and Range of Ions in Solids*. New York: Pergamon Press, 1985.
19. Ziegler J. F., Biersack J. P., Ziegler M. D. *The Stopping and Range Ions in Matter*. Chester, MD: SRIM Co., 2008.
20. Ziegler, J. F., Manoyan, J. M. "Core and Bond Values Used in SRIM." *SRIM.org*. <https://www.srim.org/SRIM/CompoundsCABTheory.htm> (accessed April 25, 2025)
21. R. A. Weller. *Scattering and Reaction Kinematics - Appendix 4*. Warrendale: Materials Research Society , 2009.
- Agostinelli, S., Allison, J., Amako, K., Apostolakis, J., Araujo, H., Arce, P., Asai, M. "Geant4 – A Simulation Toolkit." *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* 506: 250–303.

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