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Rare *bs* - decays in the covariant quark model

Abstract. Considering the surge of great interest to the problem of CP violation recently observed in a $B_s - \overline{B}_s$ system, decays of B_s to $D - \overline{D}$ and color suppressed decay $B_s \to J/\psi\ \varphi$ attracted the attention of both theorists and experimentalists. We find new values for the covariant quark model parameters (with built-in infrared confinement) in the meson sector by fitting the leptonic decay constants and a number of electromagnetic decays, and then evaluate in a parameter-free way the form factors of $B(B_s) \to P(V)$ transitions within an entire kinematic range of the momentum transfer. Our results are applied to calculating the widths of nonleptonic Bs decays to $D_S^- D_S^+$, $D_S^{*-} D_S^+ + D_S^- D_S^{*+}$ and $D_S^{*-} D_S^{*+}$. The largest contribution to $\Delta\Gamma$ for $B_s - \overline{B}_s$ system comes from these modes. The nonleptonic decay $Bs \to J/\psi\varphi$ is also considered. Though this decay mode is color suppressed it has important implications in the search for possible CP-violating novel physics effects in $B_s - \overline{B}_s$ mixing.

Key words: hadronic interactionsLagrangian, covariant quark model, heavy hadrons, multiquark states

Introduction

The covariant quark model is an effective quantum field approach to hadronic interactions based on the interaction Lagrangian between hadrons and their constituent quarks. Knowing a corresponding interpolating quark current allows calculating the matrix element of physical processes in a consistent way. A distinctive feature of this approach is that the multiquark states, such as baryons (three quarks), tetraquarks (four quarks), etc., can be considered and described as rigorously as the simplest quark–anti quark systems (mesons). The coupling constants between hadrons and their interpolating quark currents are determined from the compositeness condition $Z_H = 0$ proposed in [1, 2] and used further in numerous subfields of particle physics [3]. Here Z_H is a renormalization constant of the hadron wave function. The matrix elements of physical processes are determined by a set of associated quark diagrams, which are constructed according to $1/N_c$ expansion. In the covariant quark model an infrared cutoff is effectively introduced in the space of Fock–Schwinger parameters, which are integrated out in the expressions for the matrix elements. Such a procedure allows one to eliminate all the threshold singularities associated with quark production and thereby ensures quark confinement. The model has no ultraviolet divergences due to vertex hadron–quark form factors, which describe a nonlocal structure of hadrons. The covariant quark model features a few free parameters: a mass of constituent quarks, an infrared cutoff parameter that characterizes confinement region, and parameters that describe an effective size of hadrons.

The study of heavy quark physics provides a unique opportunity to determine the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements. Such investigations also enable progressing in understanding the origin of quark flavors and mechanisms of CP-symmetry violation. In addition, one of the main goals of experiments in heavy quark physics is the search for the signs of new physics beyond the Standard Model [4]. We investigate heavy hadrons composed of either *b* or *c* quarks and their weak decays. It should be noted that the last heaviest quark *t* decays too fast to participate in the formation of sufficiently stable hadrons. The time-dependent measurements of CP-symmetry violation has recently become possible in a $B_s - \overline{B}_s$ system. Thanks to these measurements, great attention from both theorists and experimenters was drawn to the decay $B_s \to J/\psi\ \varphi$ [5, 6].

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The leitmotif of theoretical investigations in the field of heavy quarks is to separate the small distance contributions, which can be described within the framework of perturbative quantum chromodynamics (QCD), and large distance ones, which require employment of nonperturbative methods. The simplest and most popular method is a so-called "naive" factorization based on deriving the weak interaction effective Hamiltonians, which describe weak transitions of quarks and leptons. These latter Hamiltonians are in fact sets of local quark–quark and quark–lepton operators multiplied by the so-called Wilson coefficients [7], which characterize the small distance dynamics and can be estimated by using perturbative methods, in particular, operator product expansions. While nonperturbative methods are required to calculate the matrix elements of local operators between initial and final states. Here, the way in which hadrons are composed of quarks needs to be known. Technically, any matrix element of the local operator can be expressed through a set of Lorentz structures multiplied by the scalar functions, which depend on kinematic variables. These scalar functions are called form factors.

Besides a naive factorization there are more advanced methods of separating the contributions from small and large distances. They are a so-called QCD factorization and soft-collinear effective theory (SCET). Within the framework of these approaches the factorization theorems are derived, which allow systematic description of some process in terms of "soft" and "hard" matrix elements. We refer the interested reader to [8-11] and further references therein.

There are numerous theoretical approaches to calculating the necessary hadronic form factors. Let us mention some of them. It is believed that the most model independent approach is QCD sum rules on the light cone, [12, 13]. It lets the form factors be only calculated in the region of sufficiently small momentum transfers (or large recoils). The calculated form factors are then extrapolated toward the region of large momentum transfers (or small recoils) by using the pole approximations. In [14] a systematic approach to describing rare decays $B \to K^* l^+ l^-$ in the region of small recoils was developed by using the heavy quark effective theory. A detailed analysis of the decays with small recoil that exploits this approach was later carried in [15, 16]. Let us quote several model approaches to calculating the form factors, which are based on principles that differ from that the light cone sum rules leans on: Dyson–Schwinger equation in QCD [17]; constituent quark model with dispersion relations [18, 19]; relativistic quark model with potentials [20]; QCD relativistic potential model [21, 22]; QCD sum rules [23, 24].

It should be noted that within the framework of the covariant quark model we develop the hadronic form factors can be calculated within an entire kinematic range of the momentum transfers. In [23] the form factors of $B(B_s) \to P(V)$ transitions were computed within an entire kinematic range of the momentum transfer squared by using the covariant quark model with infrared confinement. As an application of the results obtained, the widths of semileptonic decays $B_S \to D_S^- D_S^+$, $D_S^{*-} D_S^+ + D_S^- D_S^{*+}$ and $B_S \to D_S^{*-} D_S^{*+}$ were evaluated. These modes give a leading contribution to the quantity $\Delta\Gamma$ in a $B_s - \overline{B}_s$ system. The color-suppressed decay $B_s \to J/\psi\varphi$ was analyzed as well. However, this decay is important in searches for possible manifestations of new physics that lead to CP violation in a $B_s - \overline{B}_s$ system.

Covariant quark model

1.1 Lagrangian and the compositeness condition, infrared confinement.

In this section we briefly describe theoretical assumptions underlying the covariant quark model. The starting point is an invariant Lagrangian describing the interaction of some hadron with its constituent quarks. Here, a hadronic state is described by the field $H(x)$ that satisfies a respective free equation of motion, while quark part is given by the interpolating quark current

$J_H(x)$ with quantum numbers of a hadron in question:

$$L_{\text{int}}(x) = g_H H(x) \cdot J_H(x) + h.c. \qquad (1)$$

In the case of simplest quark–antiquark states (mesons) the interpolating quark current is written as follows:

$$J_M(x) = \int dx_1 \int dx_2 F_M(x; x_1, x_2)\ \overline{q}_2(x_2)\ \Gamma_M\ q_1(x_1) \qquad (2)$$

The vertex function F_M effectively describes quark distribution inside a meson. In principle, it can be related with the Bethe–Salpeter amplitude, but at this point we will consider it a phenolmenological function. It follows from the requirement of translational invariance that this function must satisfy the relation $F_M(x+a, x_1+a, x_2+a)=F_M(x, x_1, x_2)$, where a stands for an arbitrary four-vector. We choose the following form for the function F_M that satisfies this condition:

$$F_M\left(x;x_1,x_2\right)=\delta\left(x-x_1\omega_1-x_2\omega_2\right)\Phi_M\left(\left(x_1-x_2\right)^2\right), \tag{3}$$

where $\omega_i=m_{q_i}/\left(m_{q_1}+m_{q_2}\right)$.

The coupling constant g_H in Eq. (1) is constrained by the so-called compositeness condition originally proposed in [25, 26] and extensively used in [27, 28]. The compositeness condition requires that the renormalization constant of the elementary meson field $H(x)$ is set to zero:

$$Z_M=1-g_M^2\Pi'_M\left(m_M^2\right)=0, \tag{4}$$

where $\Pi'_M\left(m_M^2\right)$ is the derivative of the meson mass operator.

To clarify the physical meaning of the compositeness condition in Eq. (4), we first want to remind the reader that the renormalization constant $Z_H^{1/2}$ can also interpreted as the matrix element between the physical and the corresponding bare state. The condition $Z_H=0$ implies that the physical state does not contain the bare state and is appropriately described as a bound state. The interaction Lagrangian of Eq. (1) and the corresponding free parts of the Lagrangian describe both the constituents (quarks) and the physical particles (hadrons) which are viewed as the bound states of the quarks. As a result of the interaction, the physical particle is dressed, i.e. its mass and wave function have to be renormalized. In a more familiar setting, the compositeness condition $Z_H=0$ guarantees the correct charge normalization of a charged particle at zero momentum transfer. This can be seen by using an identity relating the derivative of the freequark propagator (with loop momentum k+p) with the electromagnetic γ_μ coupling to the same propagator at zero momentum transfer. The identity reads

$$\frac{d}{dp^\mu}\frac{1}{m-\not k-\not p}=\frac{1}{m-\not k-\not p}\gamma_\mu\frac{1}{m-\not k-\not p}. \tag{5}$$

The contribution of the left-hand-side of Eq. (5) is normalized due to the compositeness condition, and, therefore, the contribution of the right-hand-side is also normalized.

The condition $Z_H=0$ also effectively excludes the constituent degrees of freedom from the space of physical states. It thereby guarantees that there is no double counting for the physical observable under consideration. The constituents exist only in virtual states. One of the corollaries of the compositeness condition is the absence of a direct interaction of the dressed charged particle with the electromagnetic field. Taking into account both the treelevel diagram and the diagrams with self-energy insertions into the external legs (i.e. the tree-level diagram times $Z_H=1$) yields a common factor Z_H, which is equal to zero. We refer the interested reader to our previous papers [27-29] where these points are discussed in more detail. In the case of pseudo scalar and vector mesons, the derivative of the meson mass operator appearing in Eq. (4) can be calculated from the one-loop two-point function given by

$$\Pi'_P\left(p^2\right)=\frac{1}{2p^2}p^\alpha N_c\int\frac{d^4k}{(2\pi)^4 i}\tilde\Phi_P^2\left(-k^2\right)tr\left[\gamma^5 S_1(k+\omega_1 p)\gamma^5 S_2\left(k-\omega_2 p\right)\right]=$$

$$=\frac{1}{2p^2}N_c\int\frac{d^4k}{(2\pi)^4 i}\tilde\Phi_P^2\left(-k^2\right)\Big\{\ \omega_1\times tr\left[\gamma^5 S_1(k+\omega_1 p)\not p S_1\left(k+\omega_1 p\right)\gamma^5 S_2\left(k-\omega_2 p\right)\right]-$$

$$-\omega_2\times tr\left[\gamma^5 S_1\left(k+\omega_1 p\right)\gamma^5 S_2\left(k-\omega_2 p\right)\not p S_2\left(k-\omega_2 p\right)\right]\Big\}\ ,$$

$$\Pi'_V\left(p^2\right)=\frac{1}{3}\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)\frac{1}{2p^2}p^\alpha\frac{d}{dp^\alpha}N_c\int\frac{d^4k}{(2\pi)^4 i}\tilde{\Phi}_V^2\left(-k^2\right)\times$$
$$\times tr\left[\gamma^\mu S_1\left(k+\omega_1 p\right)\gamma^\nu S_2\left(k-\omega_2 p\right)\right]=\frac{1}{3}\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)\frac{1}{2p^2}N_c\int\frac{d^4k}{(2\pi)^4 i}\tilde{\Phi}_V^2\left(-k^2\right) \quad (6)$$
$$\left\{\omega_1\times tr\left[\gamma^\mu S_1\left(k+\omega_1 p\right)\not{p}S_1\left(k+\omega_1 p\right)\gamma^\nu S_2\left(k-\omega_2 p\right)\right]-\right.$$
$$\left.-\omega_2\times tr\left[\gamma^\mu S_1\left(k+\omega_1 p\right)\gamma^\nu S_2\left(k-\omega_2 p\right)\not{p}S_2\left(k-\omega_2 p\right)\right]\right\}.$$

where $\Pi`_p\,(p^2)$, $\Pi`_V\,(p^2)$ is the Fourier-transform of the vertex function $\Phi_H\,((x_1-x_2)^2)$, $S_i(k)$ is the free-quark propagator given by

$$S_i\left(k\right)=\frac{1}{m_i-\not{k}} \quad (7)$$

and m_{qi} is the effective constituent quark mass m_{qi}.

For calculational convenience, we will choose a simple Gaussian form for the vertex function $\tilde{\Phi}_H\left(-k^2\right)$. One has

$$\tilde{\Phi}_H\left(-k^2\right)=\exp\left(k^2/\Lambda_H^2\right), \quad (8)$$

where the parameter Λ_H characterizes the size of the respective bound state meson H.

Consider an arbitrary Feynman diagram consisting of n quark propagators S, l loops with momentum integration variables k, and m vertices with Gaussian vertex functions Φ. In Minkowski space this diagram can be represented in the form:

$$\Pi\left(p_1,\ldots,p_m\right)=\int\left[d^4k\right]^l\prod_{i_1=1}^{m}\Phi_{i_1+n}\left(-K_{i_1+n}^2\right)\prod_{i_3=1}^{n}S_{i_3}\left(\tilde{k}_{i_3}+\upsilon_{i_3}\right), \quad (9)$$

$$K_{i_1+n}^2=\sum_{i_2}\left(\tilde{k}_{i_1+n}^{(i_2)}+\upsilon_{i_1+n}^{(i_2)}\right)^2.$$

Further, we use the Fock–Schwinger representation for a free quark propagator:

$$S\left(k\right)=\left(m+\not{k}\right)\int_0^\infty d\beta\, e^{-\beta\left(m^2-k^2\right)}. \quad (10)$$

The loop momentum now appears in the exponent which allows one to deal very efficiently with tensor loop integrals by converting loop moment into derivatives via the identity

$$k_i^\mu e^{2kr}=\frac{1}{2}\frac{\partial}{\partial r_{i\nu}}e^{2kr} \quad (11)$$

followed by the replacement of variables $\beta_i=t\alpha_i$:

We have written a FORM [30] program that achieves the necessary commutations of the differential operators in a very efficient way. After doing the loop integration, one obtains

$$\Pi=\int_0^\infty d^n\beta\, F\left(\beta_1,\ldots,\beta_n\right), \quad (12)$$

where F is an integrand obtained this way. It is convenient to proceed to simplex integration by introducing unity into the integrand

$$1=\int_0^\infty dt\,\delta\left(t-\sum_{i=1}^{n}\beta_i\right) \quad (13)$$

$$\Pi=\int_0^\infty dt\, t^{n-1}\int_0^1 d^n\alpha\,\delta\left(1-\sum_{i=1}^{n}\alpha_i\right)F\left(t\alpha_1,\ldots,t\alpha_n\right). \quad (14)$$

As a result, there are n integrations: $(n-1)$ ones done over dimensionless variables α running a simplex, and one integration over t variable, which has a dimension of an inverse mass squared and takes the values ranging from zero to infinity. If kinematic variables corresponding to a given diagram are such that a branch point appears, then the integral in (14) starts do diverge at $t \to \infty$. However, if the integration is cut off from the above, then this guarantees the absence of any threshold singularities in a given diagram, because an integral obtained this way converges absolutely for any set of kinematic variables:

$$\Pi^c = \int_0^{1/\lambda^2} dt\, t^{n-1} \int d^n\alpha\, \delta\left(1-\sum_{i=1}^{n}\alpha_i\right) F\left(t\alpha_1,...,t\alpha_n\right) . \quad (15)$$

The cutoff parameter λ is called an infrared one.

1.2 Model parameters

First, let us define the number of free parameters in the covariant quark model in the case of mesons considered as the quark–antiquark states. For a given meson H_i there is the coupling constant g_{H_i} parameter Λ_{H_i} two out of four possible constituent quark masses m_{qj} $(m_u=m_d,\ m_s,\ m_c,\ m_b)$, and universal infrared cutoff parameter λ (confinement). It is easy to find that in the case of n_H mesons there are $2n_H+5$ free parameters. The compositeness condition imposes n_H constraints on the number of model parameters, which can symbolically be written as follows:

$$f_{H_i}\left(g_{H_i},\Lambda_{H_i},m_{q_i},\lambda\right)=1. \quad (16)$$

The constraint (16) can be used, e.g., to eliminate the coupling parameter g_{H_i} from the set of parameters. The remaining parameters are determined by a fit to experimental data. An obvious choice is to fit the model parameters to the experimental values of the leptonic decay constants.

At last, the experimental value for f_{K*} is found from the measured decay width of $\tau \to K^*\nu_\tau$. The results of the least square fits are presented in Tables 1 and 2. As is seen, agreement between fitted quantities and their experimental values is quite satisfactory.

The best fit results were obtained for the free model parameters given just below in (17-19):

m_u	m_s	m_c	m_b	λ	
0.235	0.424	2.16	5.09	0.181	Гэв

(17)

Λ_π	Λ_K	Λ_D	Λ_{D_s}	Λ_B	Λ_{B_s}	Λ_{B_c}	Λ_ρ	
0.87	1.04	1.47	1.57	1.88	1.95	2.42	0.61	Гэв

(18)

Λ_ω	Λ_φ	$\Lambda_{J/\psi}$	Λ_{K*}	Λ_{D*}	$\Lambda_{D_s^*}$	Λ_{B*}	$\Lambda_{B_s^*}$	
0.47	0.88	1.48	0.72	1.16	1.17	1.72	1.71	Гэв

(19)

Table 1 – Results obtained from the fit of the leptonic constants f_H (MeV).

	Model	Experiment	References		Model	Experiment	References
f_π	128.7	130.4±0.2	[31]	f_ω	198.5	198±2	[31]

f_K	156.1	156.1±0.8	[31]	f_φ	228.2	227±2	[31]
f_ω	205.9	206.7±8.9	[31]	$f_{J/\psi}$	415.0	415±7	[31]
f_{D_s}	257.5	257.5±6.1	[31,]	f_{K^*}	213.7	217±7	[31]
f_B	191.1	192.8±9.9	[32]	f_{D^*}	243.3	245±20	[34]
f_{B_s}	234.9	238.8±9.5	[32]	$f_{D_s^*}$	272.0	272±26	[34]
f_{B_c}	489.0	489±5	[33]	f_{B^*}	196.0	196±44	[34]
f_ρ	221.1	221±1	[31]	$f_{B_s^*}$	229.0	229±46	[34]

Table 2 – Results obtained from the fit of the widths of basic radiative decays (in keV).

Process	Model	Experiment [31]	Process	Model	Experiment [31]
$\pi^0 \to \gamma\gamma$	5.06×10^{-3}	$(7.7\pm0.4)\times10^{-3}$	$K^{*\pm} \to K^\pm\gamma$	55.1	50±5
$\eta_c \to \gamma\gamma$	1.61	1.8±0.8	$K^{*0} \to K^0\gamma$	116	116±10
$\rho\pm \to \pi^\pm\gamma$	76.0	67±7	$D^{*\pm} \to D^\pm\gamma$	1.22	1.5±0.5
$\omega \to \pi^0\gamma$	672	703±25	$J/\psi \to \eta_c\gamma$	1.43	1.58±0.37

Transition form factors

With all the model parameters being fixed, we will calculate the form factors, which describe transitions of heavy $B(B_s)$ mesons to light ones; for example, B, $Bs\to$ π, K, ρ, K^*, φ. These quantities are of great interest because of the need to know them in order to describe semileptonic, nonleptonic, and rare decays of B and B_s mesons. As has already been noted in the introduction, they were calculated by applying the light cone sum rules in the region of low momentum transfers followed by their extrapolation to an entire kinematic range.

First of all, we define the form factors for pseudoscalar–pseudoscalar and pseudoscalar–vector transitions:

$$\left\langle P'_{[\bar{q}_3 q_2]}(p_2)\ \left|\bar{q}_2\, O^\mu q_1\right|\ P_{[\bar{q}_3 q_1]}(p_1)\right\rangle = N_c g_P g_{P'} \int \frac{d^4k}{(2\pi)^4 i} \tilde{\Phi}_P\left(-(k+\omega_{13})^2\right) \times$$
$$\times \tilde{\Phi}_{P'}\left(-(k+\omega_{23})^2\right) tr\left[\ O^\mu S_1(k+p_1)\ \gamma^5 S_3(k)\ \gamma^5 S_2(k+p_2)\ \right] = F_+\left(q^2\right)\ P^\mu + F_-\left(q^2\right)\ q^\mu, \tag{20}$$

$$\left\langle P'_{[\bar{q}_3 q_2]}(p_2)\ \left|\bar{q}_2\ \left(\sigma^{\mu\nu} q_\nu\right)\ q_1\right|\ P_{[\bar{q}_3 q_1]}(p_1)\right\rangle = N_c g_P g_{P'} \int \frac{d^4k}{(2\pi)^4 i} \tilde{\Phi}_P\left(-(k+\omega_{13})^2\right)$$
$$\tilde{\Phi}_{P'}\left(-(k+\omega_{23})^2\right) tr\left[\ \sigma^{\mu\nu} q_\nu S_1(k+p_1)\ \gamma^5 S_3(k)\ \gamma^5 S_2(k+p_2)\ \right] =$$
$$= \frac{i}{m_1+m_2}\left(q^2 P_\mu - q\cdot P\, q^\mu\right)\ F_T\left(q^2\right)\ , \tag{21}$$

$$\left\langle V\left(p_2,\xi_2\right)_{[\bar{q}_3 q_2]}\left|\bar{q}_2\ O^{\mu} q_1\right| P_{[\bar{q}_3 q_1]}\left(p_1\right)\right\rangle = N_c g_P g_V \int \frac{d^4k}{\left(2\pi\right)^4 i}\tilde{\Phi}_P\left(-\left(k+\omega_{13}\right)^2\right)$$
$$\times\tilde{\Phi}_V\left(-\left(k+\omega_{23}\right)^2\right) tr\left[O^{\mu}S_1\left(k+p_1\right)\ \gamma^5 S_3\left(k\right)\xi_2^{\dagger}S_2\left(k+p_2\right)\right] = \frac{\xi_{\nu}^{\dagger}}{m_1+m_2} \tag{22}$$
$$\times\left(-g^{\mu\nu}P\cdot q\ A_0\left(q^2\right)+P^{\mu}P^{\nu}A_+\left(q^2\right)++q^{\mu}P^{\nu}A_-\left(q^2\right)+i\varepsilon^{\mu\nu\alpha\beta}P_{\alpha}\ q_{\beta}\ V\left(q^2\right)\right)\ ,$$

$$\left\langle V\left(p_2,\xi_2\right)_{[\bar{q}_3 q_2]}\ \left|\bar{q}_2\left(\sigma^{\mu\nu}q_{\nu}\left(1+\gamma^5\right)\right)q_1\right|\ P_{[\bar{q}_3 q_1]}\left(p_1\right)\right\rangle = N_c g_P g_V \int \frac{d^4k}{\left(2\pi\right)^4 i}\ \tilde{\Phi}_P\left(-\left(k+\omega_{13}\right)^2\right)$$
$$\times\tilde{\Phi}_V\left(-\left(k+\omega_{23}\right)^2\right) tr\left[\left(\sigma^{\mu\nu}q_{\nu}\left(1+\gamma^5\right)\right)S_1\left(k+p_1\right)\gamma^5 S_3\left(k\right)\xi_2^{\dagger}S_2\left(k+p_2\right)\right] = \tag{23}$$
$$= \frac{\xi_{\nu}^{\dagger}}{m_1+m_2}\left(-g^{\mu\nu}P\cdot q\ A\left(q^2\right)+P^{\mu}P^{\nu}A_+\left(q^2\right)+q^{\mu}P^{\nu}A_-\left(q^2\right)+i\varepsilon^{\mu\nu\alpha\beta}P_{\alpha}q_{\beta}V\left(q^2\right)\right) =$$
$$= \xi_{\nu}^{\dagger}\left(-\left(g^{\mu\nu}-q^{\mu}q^{\nu}/q^2\right)P\cdot q a_0\left(q^2\right)+\left(P^{\mu}P^{\nu}-q^{\mu}P^{\nu}P\cdot q/q^2\right)a_+\left(q^2\right)+i\varepsilon^{\mu\nu\alpha\beta}P_{\alpha}q_{\beta}g\left(q^2\right)\right)\ .$$

We collect in Table 3 all our results for the form factors at the point $q_2 = 0$ in which the recoil is maximal and also present those obtained by using other approaches, just for comparison.

Conclusions

A brief review of theoretical foundations underlying the covariant quark model is given. The covariant Lagrangians describing an effective interaction of hadrons with their constituent quarks are discussed. The compositeness condition, which is an effective tool in describing the bound states in quantum field theory, is presented. The notion of infrared confinement, which ensures the absence of any threshold singularities associated with quark production, is introduced in the amplitudes of physical processes.

Various transition form factors of $B(B_s)$-mesons are calculated in the entire kinematic range of the transferred momentum squared. As an application, these form factors are used to calculate the matrix elements and widths of two-particle nonleptonic decays of B_s-meson.

Table 3 – Form factors at $q_2 = 0$, calculated in various approaches.

	This paper	[8]	[9]	[13]	[19]	[14-15]	[16]	[35]
$F_+^{B\pi}(0)$	0.29	0.258±0.031			0.24±0.03	0.29	0.22	0.27
$F_+^{BK}(0)$	0.42	0.335±0.042	0.31±0.04	0.30±0.06	0.25±0.03	0.36	…	0.36
$F_T^{B\pi}(0)$	0.27	0.253±0.028	0.21±0.04	0.25±0.05	…	0.28	…	…
$F_T^{B\pi}(0)$	0.40	0.359±0.038	0.27±0.04	0.32±0.06	0.14±0.03	0.35	. . .	0.34
$V^{B\rho}(0)$	0.28	0.324±0.029	0.32±0.10	0.31±0.06	…	0.31	0.30	…
$V^{BK^*}(0)$	0.36	0.412±0.045	0.39±0.11	0.37±0.07	0.47±0.03	0.44	…	…
$V^{B_s\varphi}(0)$	0.32	0.434±0.035	…	…	0.434±0.035	…	…	…
$A_1^{B\rho}(0)$	0.26	0.240 ±0.024	0.24 ±0.08	0.24 ±0.05	…	0.26	0.27	…
$A_1^{BK^*}(0)$	0.33	0.290±0.036	0.30±0.08	0.29±0.06	0.37±0.03	0.36	…	…
$A_1^{B_s\varphi}(0)$	0.29	0.311±0.029	…	…	…	…	…	…
$A_2^{B\rho}(0)$	0.24	0.221 ±0.023	0.21± 0.09	0.25 ±0.05	…	0.24	0.28	…

$A_2^{BK^*}(0)$	0.32	0.258±0.035	0.26±0.08	0.30±0.06	0.40±0.03	0.32	…	…
$A_2^{B_s\varphi}(0)$	0.28	0.234± 0.028	…	…	…	…	…	…
$T_1^{B\rho}(0)$	0.25	0.268± 0.021	0.28± 0.09	0.26± 0.05	…	0.27	…	…
$T_1^{BK^*}(0)$	0.33	0.332±0.037	0.33±0.10	0.30±0.06	0.19±0.03	0.39	…	…
$T_1^{B_s\varphi}(0)$	0.28	0.349 ±0.033	…	…	…	…	…	…

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